c $rc_l = 4$ en l is evenwijdig met de raaklijnen, dus $f'(x) = 4$.
$f'(x) = 4$ geeft $x^2 - x - 2 = 4$
$x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x = -2 \lor x = 3$
$x_B = -2$ en $y_B = f(-2) = \frac{1}{3}$, dus $B(-2, \frac{1}{3})$.
$x_C = 3$ en $y_C = f(3) = -\frac{1}{2}$, dus $C(3, -\frac{1}{2})$.

bladzijde 172

29 **a** $f(x) = (x^2 + 2)(1 - x) = x^2 - x^3 + 2 - 2x = -x^3 + x^2 - 2x + 2$
geeft $f'(x) = -3x^2 + 2x - 2$
$f'(2) = -3 \cdot 2^2 + 2 \cdot -2 = -10$, dus $rc_k = -10$.
$\left.\begin{array}{l} k\colon y = -10x + b \\ f(2) = -6, \text{ dus } A(2, -6) \end{array}\right\} \begin{array}{l} -6 = -10 \cdot 2 + b \\ -6 = -20 + b \\ 14 = b \end{array}$
Dus $k\colon y = -10x + 14$.

b $rc_k = -10$ en k is evenwijdig met de raaklijn, dus $f'(x) = -10$.
$f'(x) = -10$ geeft $-3x^2 + 2x - 2 = -10$
$-3x^2 + 2x + 8 = 0$
$D = 2^2 - 4 \cdot -3 \cdot 8 = 100$, dus $\sqrt{D} = \sqrt{100} = 10$
$x = \dfrac{-2 - 10}{-6} = 2 \lor x = \dfrac{-2 + 10}{-6} = -1\frac{1}{3}$
Dus $x_B = -1\frac{1}{3}$.

30 **a** $s = 0{,}06t^3 + 1{,}2t^2$ geeft $v = 0{,}18t^2 + 2{,}4t$
Op $t = 4$ is de snelheid $v = 0{,}18 \cdot 4^2 + 2{,}4 \cdot 4 = 12{,}48$ m/s.
Op $t = 6$ is de snelheid $v = 0{,}18 \cdot 6^2 + 2{,}4 \cdot 6 = 20{,}88$ m/s.

b 100 km/uur $= \dfrac{100}{3{,}6}$ m/s
Voer in $y_1 = 0{,}18x^2 + 2{,}4x$ en $y_2 = 100/3{,}6$.
De optie intersect geeft $x \approx 7{,}43$.
Dus na ongeveer 7,43 seconden is de snelheid 100 km/uur.

c Na 8 seconden is $s = 0{,}06 \cdot 8^3 + 1{,}2 \cdot 8^2 = 107{,}52$ m.
Op $t = 8$ is de snelheid $v = 0{,}18 \cdot 8^2 + 2{,}4 \cdot 8 = 30{,}72$ m/s.
$300 - 107{,}52 = 192{,}48$ m
$\dfrac{192{,}48}{30{,}72} \approx 6{,}3$, dus na ongeveer $8 + 6{,}3 = 14{,}3$ seconden heeft de motor 300 meter afgelegd.

4 Algebra en meetkunde

31 **a** $\left(2a + \sqrt{3}\right)^2 = 4a^2 + 4a\sqrt{3} + 3$

b $\left(a + 2\sqrt{3}\right)\left(a - 2\sqrt{3}\right) = a^2 - 12$

c $\left(2\sqrt{2} + 3\sqrt{8}\right)^2 = 8 + 12\sqrt{16} + 72 = 8 + 48 + 72 = 128$

d $\sqrt{\tfrac{1}{2}} + 6\sqrt{32} = \dfrac{\sqrt{1}}{\sqrt{2}} + 6\sqrt{16 \cdot 2} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} + 6 \cdot 4\sqrt{2} = \dfrac{\sqrt{2}}{2} + 24\sqrt{2}$
$= \frac{1}{2}\sqrt{2} + 24\sqrt{2} = 24\frac{1}{2}\sqrt{2}$

e $\dfrac{\sqrt{2}+1}{\sqrt{2}-1} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \cdot \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \dfrac{\left(\sqrt{2}+1\right)\left(\sqrt{2}+1\right)}{2-1} = \dfrac{2+2\sqrt{2}+1}{1} = 3 + 2\sqrt{2}$

f $\dfrac{5}{\sqrt{3}+1} = \dfrac{5}{\sqrt{3}+1} \cdot \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{5\left(\sqrt{3}-1\right)}{3-1} = \dfrac{5\sqrt{3}-5}{2} = 2\frac{1}{2}\sqrt{3} - 2\frac{1}{2}$

g $\dfrac{\sqrt{8}+\sqrt{12}}{2\sqrt{3}} = \dfrac{\sqrt{8}+\sqrt{12}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{(\sqrt{8}+\sqrt{12})\cdot\sqrt{3}}{6} = \dfrac{\sqrt{24}+\sqrt{36}}{6} = \dfrac{\sqrt{4\cdot 6}+6}{6} = \dfrac{2\sqrt{6}+6}{6} = \tfrac{1}{3}\sqrt{6}+1$

h $(2a+3\sqrt{2})(2a+2\sqrt{3}) = 4a^2+4a\sqrt{3}+6a\sqrt{2}+6\sqrt{6}$

32 **a** $3x+\dfrac{6}{2x-1} = \dfrac{3x(2x-1)}{2x-1}+\dfrac{6}{2x-1} = \dfrac{6x^2-3x}{2x-1}+\dfrac{6}{2x-1} = \dfrac{6x^2-3x+6}{2x-1}$

b $\dfrac{2x-1}{x+2}-\dfrac{x+2}{x-4} = \dfrac{(2x-1)(x-4)}{(x+2)(x-4)}-\dfrac{(x+2)(x+2)}{(x+2)(x-4)}$

$= \dfrac{2x^2-8x-x+4}{(x+2)(x-4)}-\dfrac{x^2+4x+4}{(x+2)(x-4)} = \dfrac{2x^2-9x+4-(x^2+4x+4)}{(x+2)(x-4)} = \dfrac{x^2-13x}{(x+2)(x-4)}$

c $\dfrac{a^2}{2a+5}+\dfrac{a^4}{a-3} = \dfrac{a^2(a-3)}{(2a+5)(a-3)}+\dfrac{a^4(2a+5)}{(2a+5)(a-3)} = \dfrac{a^3-3a^2}{(2a+5)(a-3)}+\dfrac{2a^5+5a^4}{(2a+5)(a-3)} = \dfrac{2a^5+5a^4+a^3-3a^2}{(2a+5)(a-3)}$

d $\dfrac{3x^2+6x}{x^2+8x+12} = \dfrac{3x(x+2)}{(x+2)(x+6)} = \dfrac{3x}{x+6}$

e $\dfrac{x^4-9x^2+8}{x^4-1} = \dfrac{(x^2-1)(x^2-8)}{(x^2-1)(x^2+1)} = \dfrac{x^2-8}{x^2+1}$

f $\dfrac{a^6-5a^3+4}{6a^3-24} = \dfrac{(a^3-1)(a^3-4)}{6(a^3-4)} = \dfrac{a^3-1}{6} = \tfrac{1}{6}a^3-\tfrac{1}{6}$

33 **a** $\dfrac{1}{x+1}+\dfrac{3}{2x+1} = \dfrac{8}{15}$

$\dfrac{2x+1}{(x+1)(2x+1)}+\dfrac{3(x+1)}{(x+1)(2x+1)} = \dfrac{8}{15}$

$\dfrac{2x+1}{(x+1)(2x+1)}+\dfrac{3x+3}{(x+1)(2x+1)} = \dfrac{8}{15}$

$\dfrac{5x+4}{2x^2+3x+1} = \dfrac{8}{15}$

kruiselings vermenigvuldigen geeft

$(2x^2+3x+1)\cdot 8 = (5x+4)\cdot 15$
$16x^2+24x+8 = 75x+60$
$16x^2-51x-52 = 0$
$D = (-51)^2-4\cdot 16\cdot -52 = 5929$, dus $\sqrt{D} = \sqrt{5929} = 77$
$x = \dfrac{51-77}{32} = -\tfrac{13}{16} \vee x = \dfrac{51+77}{32} = 4$
voldoet voldoet

b $\dfrac{x^2-4}{x^2+4x+4} = 2x$

$\dfrac{(x-2)(x+2)}{(x+2)(x+2)} = 2x$

$\dfrac{x-2}{x+2} = \dfrac{2x}{1}$

kruiselings vermenigvuldigen geeft
$x-2 = (x+2)\cdot 2x$
$x-2 = 2x^2+4x$
$-2x^2-3x-2 = 0$
$2x^2+3x+2 = 0$
$D = 3^2-4\cdot 2\cdot 2 = -7$
geen oplossingen

34 **a** $x^4 \cdot \sqrt[3]{x} = x^4 \cdot x^{\frac{1}{3}} = x^{4\frac{1}{3}}$

b $\dfrac{x^{-3}}{x^2} = x^{-5}$

c $x \cdot \sqrt{\dfrac{1}{x^5}} = x \cdot \sqrt{x^{-5}} = x^1 \cdot x^{-2\frac{1}{2}} = x^{-1\frac{1}{2}}$

d $\dfrac{1}{x} \cdot \left(\sqrt[4]{x^3}\right)^8 = x^{-1} \cdot \left(x^{\frac{3}{4}}\right)^8 = x^{-1} \cdot x^6 = x^5$

e $\dfrac{x^3 \cdot x^{-5}}{\sqrt{x}} = \dfrac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2\frac{1}{2}}$

f $\left(x\sqrt{x}\right)^{-3} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{-3} = \left(x^{1\frac{1}{2}}\right)^{-3} = x^{-4\frac{1}{2}}$

35 **a** $v = 60$ en $T = -20$ geeft $F = (2000 - 16{,}3 \cdot 60)\ (-5 - -20)^{-1{,}668} \approx 11{,}2$
Dus ongeveer 11 minuten.

b $F = 20$ en $T = -18$ invullen geeft $20 = (2000 - 16{,}3v)(-5 - -18)^{-1{,}668}$
$20 \approx (2000 - 16{,}3v) \cdot 0{,}0139$
$1442{,}4 \approx 2000 - 16{,}3v$
$16{,}3v \approx 557{,}6$
$v \approx 34{,}2$
De windsnelheid is ongeveer 34 km/uur.

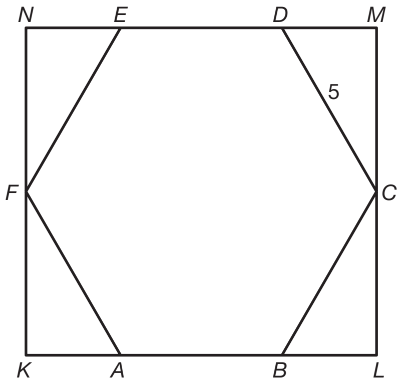
c De wedstrijd duurt $\dfrac{10}{40}$ uur = 0,25 uur = 15 minuten, dus $F = 15$.
$F = 15$ en $v = 40$ geeft $15 = \left(2000 - 16{,}3 \cdot 40\right)\left(-5 - T\right)^{-1{,}668}$
$15 = 1348 \cdot \left(-5 - T\right)^{-1{,}668}$
$\dfrac{15}{1348} = \left(-5 - T\right)^{-1{,}668}$
$-5 - T = \left(\dfrac{15}{1348}\right)^{-\frac{1}{1{,}668}}$
$-5 - T \approx 14{,}8$
$-T \approx 19{,}8$
$T \approx -19{,}8$
Bij een temperatuur van –20 ºC of lager is het niet verantwoord om de wedstrijd door te laten gaan.

36 **a** $m = 40$ en $A = 136$ geeft $136 = a \cdot 40^{0{,}67}$
$a = \dfrac{136}{40^{0{,}67}} \approx 11{,}5$

b $m = 275$ geeft $A = 11{,}5 \cdot 275^{0{,}67} \approx 496$ dm²

c $A = 1{,}16$ geeft $1{,}16 = 11{,}5 \cdot m^{0{,}67}$
$m^{0{,}67} = \dfrac{1{,}16}{11{,}5}$
$m = \left(\dfrac{1{,}16}{11{,}5}\right)^{\frac{1}{0{,}67}} \approx 0{,}0326$
Het gewicht is dus ongeveer 0,0326 kg ≈ 33 gram.

37

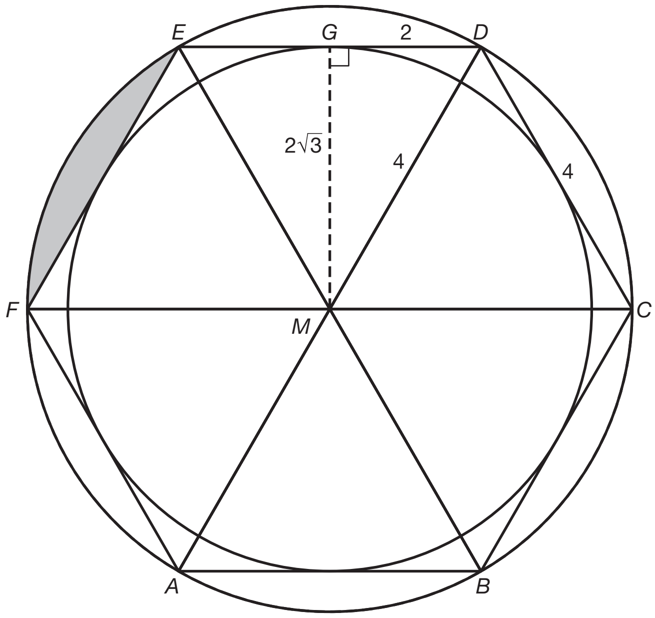


In $\triangle MDC$ is $\angle M = 90^\circ$, $\angle D = 60^\circ$ en $DC = 5$, dus $MD = 2\frac{1}{2}$ en $MC = 2\frac{1}{2}\sqrt{3}$.

Dus $MN = 2\frac{1}{2} + 5 + 2\frac{1}{2} = 10$ en $LM = 2\frac{1}{2}\sqrt{3} + 2\frac{1}{2}\sqrt{3} = 5\sqrt{3}$.

Dit geeft $O(KLMN) = 10 \cdot 5\sqrt{3} = 50\sqrt{3}$.

38



O(gekleurd) = O(omgeschreven cirkel) – O(ingeschreven cirkel) – 3 · O(grijs)

O(grijs) = $\frac{1}{6}$ O(omgeschreven cirkel) – $O(\triangle DME)$

$\angle DME = \dfrac{360^\circ}{6} = 60^\circ$, dus

$\triangle DME$ is een gelijkzijdige driehoek met zijde 4.

Uit $\angle MDG = 60^\circ$ en $DG = 2$ volgt $GM = 2\sqrt{3}$.

$O(\text{grijs}) = \frac{1}{6} \cdot \pi \cdot 4^2 - \frac{1}{2} \cdot 4 \cdot 2\sqrt{3} = 2\frac{2}{3}\pi - 4\sqrt{3}$

$O(\text{gekleurd}) = \pi \cdot 4^2 - \pi \cdot \left(2\sqrt{3}\right)^2 - 3 \cdot \left(2\frac{2}{3}\pi - 4\sqrt{3}\right)$

$= 16\pi - 12\pi - 8\pi + 12\sqrt{3} = 12\sqrt{3} - 4\pi$

39 Stel $QR = x$, dan is $\left.\begin{array}{l} DR + QC = 5 - x \\ DR = QC \end{array}\right\} DR = \frac{1}{2}(5 - x)$

$AS = QR$, dus $DS = 5 - x$.

$O(\triangle DRS) \;\; = \frac{1}{2} \cdot DR \cdot DS = \frac{1}{2} \cdot \frac{1}{2}(5 - x) \cdot (5 - x) = \frac{1}{4}(5 - x)^2$

$O(\triangle CPQ) = O(\triangle DRS) = \frac{1}{4}(5 - x^2)$

$O(ABPQRS) = O(ABCD) - O(\triangle DRS) - O(\triangle CPQ)$

$= 5^2 - \frac{1}{4}(5 - x)^2 - \frac{1}{4}(5 - x)^2$

$= 25 - \frac{1}{2}(5 - x)^2$

$O(ABPQRS) = 15$ geeft $25 - \frac{1}{2}(5 - x)^2 = 15$

$10 = \frac{1}{2}(5 - x)^2$

$(5 - x)^2 = 20$

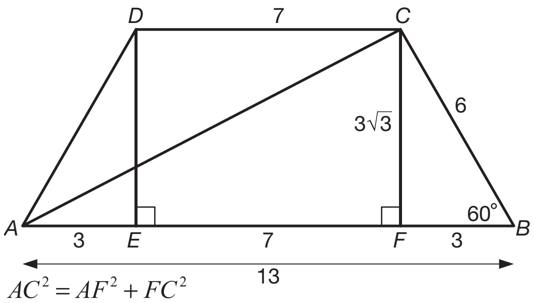
$5 - x = \sqrt{20} \;\vee\; 5 - x = -\sqrt{20}$

$x = 5 - \sqrt{20} \;\vee\; x = 5 + \sqrt{20}$

voldoet niet

Dus $AS = 5 - \sqrt{20} = 5 - \sqrt{4 \cdot 5} = 5 - 2\sqrt{5}$.

40 **a**



$AC^2 = AF^2 + FC^2$

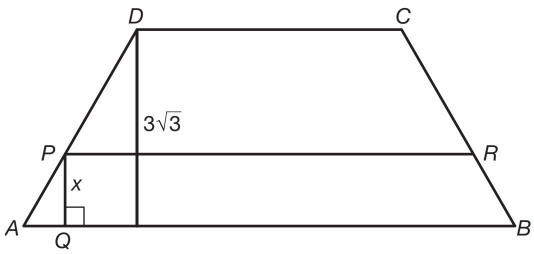
$AC^2 = 10^2 + \left(3\sqrt{3}\right)^2$

$AC^2 = 127$, dus $AC = \sqrt{127}$

b omtrek = 13 + 6 + 7 + 6 = 32

oppervlakte $= \frac{1}{2}\left(13 + 7\right) \cdot 3\sqrt{3} = 30\sqrt{3}$

c



Teken PQ loodrecht op AB en stel $PQ = x$.

Uit $\angle A = 60^\circ$ en $PQ = x$ volgt $AQ = \dfrac{x}{\sqrt{3}}$ en $PR = AB - 2 \cdot AQ = 13 - \dfrac{2x}{\sqrt{3}}$.

$$\left.\begin{array}{l} O(ABRP) = \frac{1}{2}\left(13 + 13 - \dfrac{2x}{\sqrt{3}}\right) \cdot x \\ O(ABRP) = \frac{1}{2} \cdot O(ABCD) = \frac{1}{2} \cdot 30\sqrt{3} = 15\sqrt{3} \end{array}\right\} \frac{1}{2}x\left(26 - \dfrac{2x}{\sqrt{3}}\right) = 15\sqrt{3}$$

Voer in $y_1 = \frac{1}{2}x\left(26 - \dfrac{2x}{\sqrt{3}}\right)$ en $y_2 = 15\sqrt{3}$.

De optie intersect geeft $x \approx 2{,}22$.

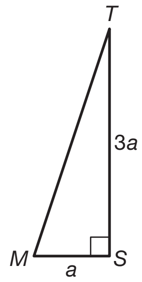
Dus de afstand tussen AB en PR is ongeveer 2,22.

bladzijde 175

41 **a** In $\triangle MST$ is $MT^2 = a^2 + (3a)^2$

$MT^2 = a^2 + 9a^2$

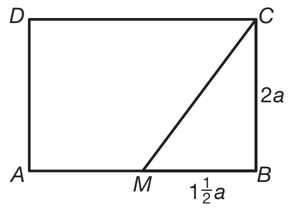
$MT^2 = 10a^2$, dus $MT = \sqrt{10a^2} = \sqrt{a^2 \cdot 10} = a\sqrt{10}$



b In $\triangle BCM$ is $CM^2 = \left(1\tfrac{1}{2}a\right)^2 + \left(2a\right)^2$

$CM^2 = 2\tfrac{1}{4}a^2 + 4a^2$

$CM^2 = 6\tfrac{1}{4}a^2$, dus $CM = \sqrt{6\tfrac{1}{4}a^2} = \sqrt{\tfrac{25}{4}a^2} = 2\tfrac{1}{2}a$



c In $\triangle ABC$ is $AC^2 = (3a)^2 + (2a)^2$

$AC^2 = 9a^2 + 4a^2$

$AC^2 = 13a^2$, dus $AC = \sqrt{13a^2} = \sqrt{a^2 \cdot 13} = a\sqrt{13}$

$CU = \tfrac{1}{2} \cdot CS = \tfrac{1}{4}AC$ dus $AU = \tfrac{3}{4}AC = \tfrac{3}{4}a\sqrt{13}$

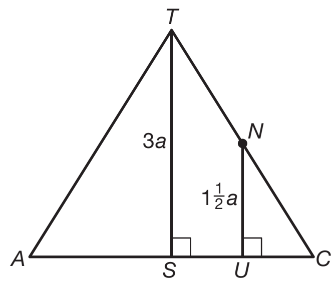
In $\triangle ANU$ is $AN^2 = AU^2 + UN^2$

$AN^2 = \left(\tfrac{3}{4}a\sqrt{13}\right)^2 + \left(1\tfrac{1}{2}a\right)^2$

$AN^2 = \tfrac{9}{16}a^2 \cdot 13 + \tfrac{9}{4}a^2$

$AN^2 = 9\tfrac{9}{16}a^2$, dus $AN = \sqrt{9\tfrac{9}{16}a^2} = \sqrt{\tfrac{153}{16}a^2}$

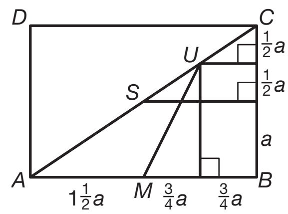
$= \sqrt{\tfrac{1}{16}a^2 \cdot 153} = \tfrac{1}{4}a\sqrt{153}$



d $MU^2 = (\tfrac{3}{4}a)^2 + (1\tfrac{1}{2}a)^2 = \tfrac{9}{16}a^2 + 2\tfrac{1}{4}a^2 = 2\tfrac{13}{16}a^2$

$MN^2 = MU^2 + UN^2 = 2\tfrac{13}{16}a^2 + (1\tfrac{1}{2}a)^2 = 2\tfrac{13}{16}a^2 + 2\tfrac{1}{4}a^2 = 5\tfrac{1}{16}a^2 = \tfrac{81}{16}a^2$,

dus $MN = \sqrt{\tfrac{81}{16}a^2} = \tfrac{9}{4}a = 2\tfrac{1}{4}a$



42 Stel $PQ = x$.

$\triangle BPQ \backsim \triangle BAC$ (snavelfiguur)

$\dfrac{PQ}{AC} = \dfrac{PB}{AB}$

$\dfrac{x}{3} = \dfrac{PB}{4}$

$PB = \dfrac{4x}{3} = 1\tfrac{1}{3}x$

$AP = 4 - 1\tfrac{1}{3}x$

$O(APQR) = AP \cdot PQ = (4 - 1\tfrac{1}{3}x)\,x = 4x - 1\tfrac{1}{3}x^2 = -1\tfrac{1}{3}x^2 + 4x$

$x_{top} = -\dfrac{b}{2a} = -\dfrac{4}{-2\tfrac{2}{3}} = 1\tfrac{1}{2}$

$x = 1\tfrac{1}{2}$ geeft $O = -1\tfrac{1}{3}(1\tfrac{1}{2})^2 + 4 \cdot 1\tfrac{1}{2} = 3$

Dus de maximale oppervlakte is 3.