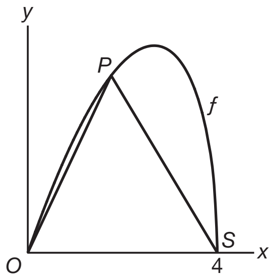
39 **a**



Stel $x_P = p$. Dit geeft $y_P = f(p) = p\sqrt{8-2p}$.

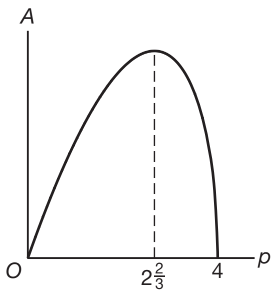
$A = O(\triangle OSP) = \frac{1}{2} \cdot OS \cdot y_P = \frac{1}{2} \cdot 4 \cdot p\sqrt{8-2p} = 2p\sqrt{8-2p}$

$$\frac{dA}{dp} = 2 \cdot \sqrt{8-2p} + 2p \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = \frac{2(8-2p)}{\sqrt{8-2p}} - \frac{2p}{\sqrt{8-2p}} = \frac{16-4p}{\sqrt{8-2p}} - \frac{2p}{\sqrt{8-2p}} = \frac{16-6p}{\sqrt{8-2p}}$$

$\frac{dA}{dp} = 0$ geeft $16 - 6p = 0$

$-6p = -16$

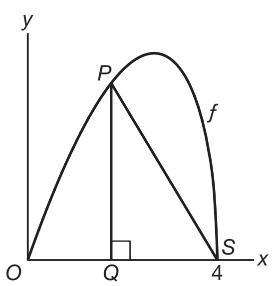
$p = 2\frac{2}{3}$



De maximale oppervlakte is

$$2 \cdot 2\tfrac{2}{3}\sqrt{8 - 2 \cdot 2\tfrac{2}{3}} = 5\tfrac{1}{3}\sqrt{2\tfrac{2}{3}} = 5\tfrac{1}{3}\sqrt{\tfrac{8}{3}} = 5\tfrac{1}{3} \cdot \frac{\sqrt{8}}{\sqrt{3}} = 5\tfrac{1}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} = 5\tfrac{1}{3} \cdot \frac{2\sqrt{6}}{3} = \tfrac{32}{9}\sqrt{6}.$$

b



Stel $x_P = p$. Dit geeft $PQ = y_P = f(p) = p\sqrt{8-2p}$ en $QS = OS - OQ = 4 - p$.

$A = O(\triangle QSP) = \frac{1}{2} \cdot QS \cdot PQ = \frac{1}{2} \cdot (4-p) \cdot p\sqrt{8-2p} = (2p - \frac{1}{2}p^2)\sqrt{8-2p}$

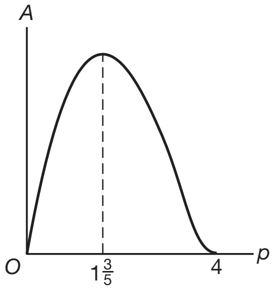
$$\frac{dA}{dp} = (2-p)\sqrt{8-2p} + (2p - \tfrac{1}{2}p^2) \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = \frac{(2-p)(8-2p)}{\sqrt{8-2p}} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}}$$

$$= \frac{16 - 12p + 2p^2}{\sqrt{8-2p}} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}} = \frac{2\frac{1}{2}p^2 - 14p + 16}{\sqrt{8-2p}}$$

$\frac{dA}{dp} = 0$ geeft $2\frac{1}{2}p^2 - 14p + 16 = 0$

$D = (-14)^2 - 4 \cdot 2\frac{1}{2} \cdot 16 = 36$, dus $\sqrt{D} = 6$

$p = \frac{14-6}{5} = 1\frac{3}{5} \vee p = \frac{14+6}{5} = 4$

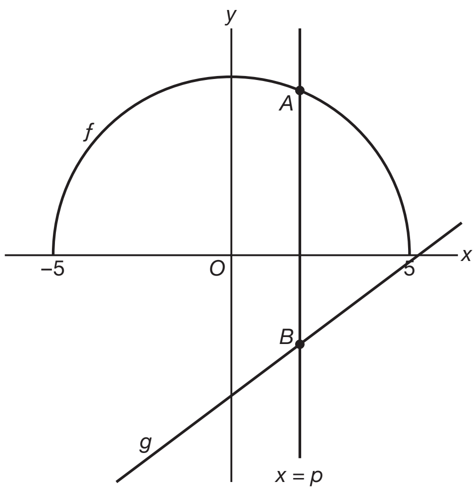


De maximale oppervlakte is

$(2 \cdot 1\frac{3}{5} - \frac{1}{2} \cdot (1\frac{3}{5})^2)\sqrt{8 - 2 \cdot 1\frac{3}{5}} = 1\frac{23}{25}\sqrt{4\frac{4}{5}} = 1\frac{23}{25}\sqrt{\frac{24}{5}}$

$= 1\frac{23}{25} \cdot \frac{\sqrt{24}}{\sqrt{5}} = 1\frac{23}{25} \cdot \frac{2\sqrt{6}}{\sqrt{5}} = 1\frac{23}{25} \cdot \frac{2\sqrt{30}}{5} = \frac{96}{125}\sqrt{30}.$

40 **a**



$L = f(p) - g(p) = \sqrt{25 - p^2} - (\frac{3}{4}p - 4) = \sqrt{25 - p^2} - \frac{3}{4}p + 4$

$\frac{\mathrm{d}L}{\mathrm{d}p} = \frac{1}{2\sqrt{25 - p^2}} \cdot -2p - \frac{3}{4} = -\frac{p}{\sqrt{25 - p^2}} - \frac{3}{4}$

$\frac{\mathrm{d}L}{\mathrm{d}p} = 0$ geeft $\frac{p}{\sqrt{25 - p^2}} = -\frac{3}{4}$

$4p = -3\sqrt{25 - p^2}$

kwadrateren geeft $16p^2 = 9(25 - p^2)$

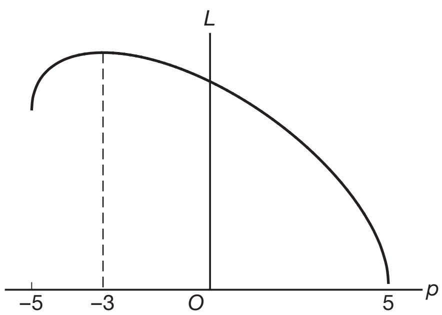
$16p^2 = 225 - 9p^2$

$25p^2 = 225$

$p^2 = 9$

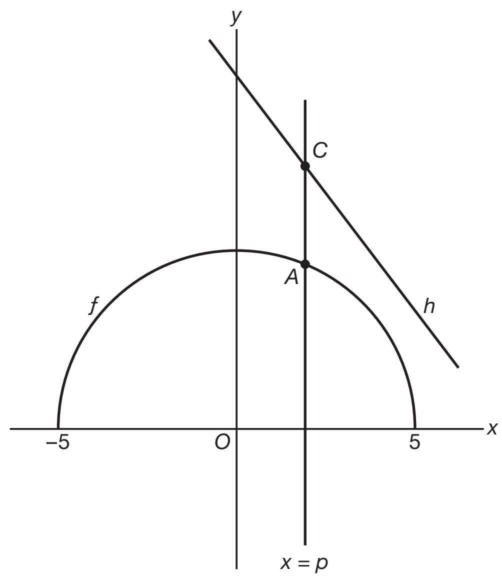
$p = -3 \vee p = 3$

voldoet   voldoet niet



De maximale lengte is $\sqrt{25 - (-3)^2} - \frac{3}{4} \cdot -3 + 4 = 10\frac{1}{4}$.

b



$L = h(p) - f(p) = -\frac{4}{3}p + 10 - \sqrt{25 - p^2}$

$\frac{dL}{dp} = -\frac{4}{3} - \frac{1}{2\sqrt{25-p^2}} \cdot -2p = -\frac{4}{3} + \frac{p}{\sqrt{25-p^2}}$

$\frac{dL}{dp} = 0$ geeft $\frac{p}{\sqrt{25-p^2}} = \frac{4}{3}$

$3p = 4\sqrt{25-p^2}$

kwadrateren geeft $9p^2 = 16(25 - p^2)$

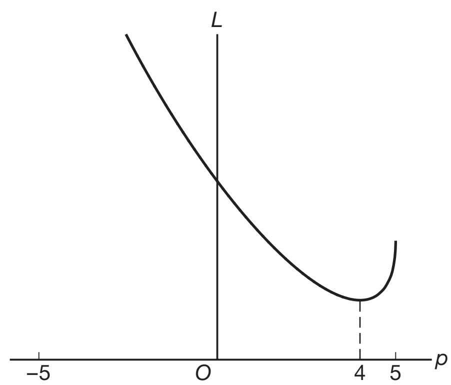
$9p^2 = 400 - 16p^2$

$25p^2 = 400$

$p^2 = 16$

$p = -4 \quad \vee \quad p = 4$

vold. niet   voldoet



De minimale lengte is $-\frac{4}{3} \cdot 4 + 10 - \sqrt{25 - 4^2} = 1\frac{2}{3}$.

bladzijde 168

41 **a** Rechte lijn op logaritmisch papier, dus $N_A = b \cdot g^t$.

Lijn door (0, 150) en (16, 1400), dus $g_{16\,\text{jaar}} = \frac{1400}{150} = 9\frac{1}{3}$ en $g_{\text{jaar}} = (9\frac{1}{3})^{\frac{1}{16}} \approx 1{,}15$.

$\left.\begin{array}{l} N_A = b \cdot 1{,}15^t \\ \text{door } (0, 150) \end{array}\right\} N_A = 150 \cdot 1{,}15^t$

Rechte lijn op logaritmisch papier, dus $N_B = b \cdot g^t$.

Lijn door (0, 800) en (18, 320), dus $g_{18\,\text{jaar}} = \frac{320}{800} = \frac{2}{5}$ en $g_{\text{jaar}} = (\frac{2}{5})^{\frac{1}{18}} \approx 0{,}95$.

$\left.\begin{array}{l} N_B = b \cdot 0{,}95^t \\ \text{door } (0, 800) \end{array}\right\} N_B = 800 \cdot 0{,}95^t$

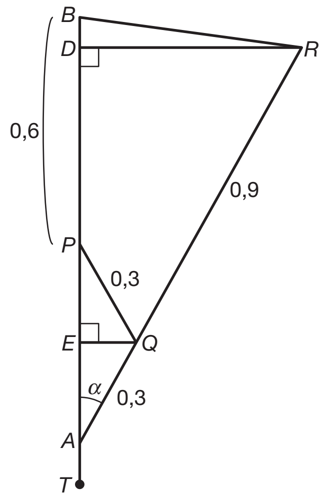
b $N = N_A + N_B = 150 \cdot 1{,}15^t + 800 \cdot 0{,}95^t$.

Voer in $y_1 = 150 \cdot 1{,}15^x + 800 \cdot 0{,}95^x$.

De optie minimum geeft $x \approx 3{,}5$ en $y \approx 913$.

Dus het minimale aantal planten van deze familie in dit natuurgebied is ongeveer 910.

42 **a**



In $\triangle ADR$ is $\sin(\alpha) = \frac{DR}{AR}$ en $\cos(\alpha) = \frac{AD}{AR}$

$\sin(\alpha) = \frac{DR}{1,2}$ $\quad\cos(\alpha) = \frac{AD}{1,2}$

$DR = 1,2\sin(\alpha)$ $\quad AD = 1,2\cos(\alpha)$

In $\triangle AEQ$ is $\cos(\alpha) = \frac{AE}{AQ}$

$\cos(\alpha) = \frac{AE}{0,3}$

$AE = 0,3\cos(\alpha)$, dus

$AP = 2 \cdot 0,3\cos(\alpha) = 0,6\cos(\alpha)$

$BD = AB - AD = 0,6 + 0,6\cos(\alpha) - 1,2\cos(\alpha) = 0,6 - 0,6\cos(\alpha)$

In $\triangle BDR$ is $BR^2 = BD^2 + DR^2$

$BR^2 = (0,6 - 0,6\cos(\alpha))^2 + (1,2\sin(\alpha))^2$

$BR^2 = 0,36 - 0,72\cos(\alpha) + 0,36\cos^2(\alpha) + 1,44\sin^2(\alpha)$

$BR^2 = 0,36 - 0,72\cos(\alpha) + 0,36\cos^2(\alpha) + 1,44(1 - \cos^2(\alpha))$

$BR^2 = 0,36 - 0,72\cos(\alpha) + 0,36\cos^2(\alpha) + 1,44 - 1,44\cos^2(\alpha)$

$BR^2 = 1,8 - 0,72\cos(\alpha) - 1,08\cos^2(\alpha)$

Dus $BR = \sqrt{1,8 - 0,72\cos(\alpha) - 1,08\cos^2(\alpha)}$.

b $BR > 1$ geeft $\sqrt{1,8 - 0,7\cos(\alpha) - 1,08\cos^2(\alpha)} > 1$

Voer in $y_1 = \sqrt{1,8 - 0,72\cos(x) - 1,08\cos^2(x)}$ en $y_2 = 1$.
De optie intersect geeft $x \approx 0,94$.
Dus de lengte van het uitgerolde doek is meer dan 1 m bij een hoek van 0,94 rad of groter.

c $y = 1,08\cos^2(\alpha) = 1,08u^2$ met $u = \cos(\alpha)$

$\frac{dy}{d\alpha} = 2,16u \cdot -\sin(\alpha) = -2,16\cos(\alpha)\sin(\alpha) = -2,16\sin(\alpha)\cos(\alpha)$

$L = BR = \sqrt{1,8 - 0,72\cos(\alpha) - 1,08\cos^2(\alpha)} = \sqrt{u}$ met $u = 1,8 - 0,72\cos(\alpha) - 1,08\cos^2(\alpha)$

$\frac{dL}{d\alpha} = \frac{1}{2\sqrt{u}} \cdot (0,72\sin(\alpha) + 2,16\sin(\alpha)\cos(\alpha)) = \frac{0,36\sin(\alpha) + 1,08\sin(\alpha)\cos(\alpha)}{\sqrt{1,8 - 0,72\cos(\alpha) - 1,08\cos^2(\alpha)}}$

$\frac{dL}{d\alpha} = 0$ geeft $0,36\sin(\alpha) + 1,08\sin(\alpha)\cos(\alpha) = 0$

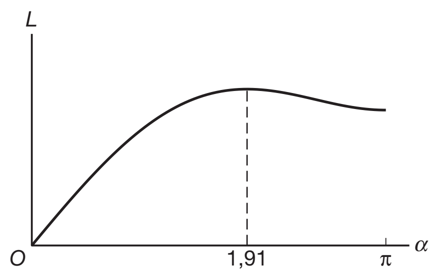
$0,36\sin(\alpha)(1 + 3\cos(\alpha)) = 0$

$0,36\sin(\alpha) = 0 \vee 1 + 3\cos(\alpha) = 0$

$\sin(\alpha) = 0 \vee 3\cos(\alpha) = -1$

$\alpha = k \cdot \pi \vee \cos(\alpha) = -\frac{1}{3}$

$\alpha = k \cdot \pi \vee \alpha \approx 1,91 + k \cdot 2\pi \vee \alpha \approx -1,91 + k \cdot 2\pi$



Dus BR is maximaal bij een hoek van ongeveer 1,91 rad.

43 **a** $f(x) = \sin(\frac{1}{3}\pi - 2x) + \cos(2x)$

$= \sin(\frac{1}{3}\pi - 2x) + \sin(2x + \frac{1}{2}\pi)$

$= 2\sin(\frac{1}{2}(\frac{1}{3}\pi - 2x + 2x + \frac{1}{2}\pi)) \cdot \cos(\frac{1}{2}(\frac{1}{3}\pi - 2x - (2x + \frac{1}{2}\pi)))$

$= 2\sin(\frac{1}{2}(\frac{5}{6}\pi))\cos(\frac{1}{2}(\frac{1}{3}\pi - 2x - 2x - \frac{1}{2}\pi))$

$= 2\sin(\frac{5}{12}\pi)\cos(\frac{1}{2}(-\frac{1}{6}\pi - 4x))$

$= 2\sin(\frac{5}{12}\pi)\cos(-\frac{1}{12}\pi - 2x)$

$= 2\sin(\frac{5}{12}\pi)\sin(-\frac{1}{12}\pi - 2x + \frac{1}{2}\pi)$

$= 2\sin(\frac{5}{12}\pi)\sin(\frac{5}{12}\pi - 2x)$

$\approx 1{,}93\sin(\frac{5}{12}\pi - 2x)$

$g(x) = 6\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$

$= 3 \cdot 2\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$

$= 3\sin(2x) + 3\sin(2x + \frac{1}{6}\pi)$

$= 3 \cdot 2\sin(\frac{1}{2}(2x + 2x + \frac{1}{6}\pi)) \cdot \cos(\frac{1}{2}(2x - 2x - \frac{1}{6}\pi))$

$= 6\sin(2x + \frac{1}{12}\pi)\cos(-\frac{1}{12}\pi)$

$= 6\cos(-\frac{1}{12}\pi)\sin(2x + \frac{1}{12}\pi)$

$\approx 5{,}80\sin(2x + \frac{1}{12}\pi)$

b De grafieken van f en g hebben dezelfde periode dus $h(x)$ is te schrijven in de vorm $y = b\sin(cx - d)$.
Omdat de periode van de grafiek van f en de grafiek van g gelijk is aan $\frac{2\pi}{2} = \pi$ is $c = \frac{2\pi}{\pi} = 2$.

Voer in $y_1 = \sin(\frac{1}{3}\pi - 2x) + \cos(2x) + 6\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$.
De optie maximum geeft $x \approx 0{,}49$ en $y \approx 6{,}11$.
De optie zero (TI) of ROOT (Casio) geeft $x \approx 2{,}850$.
Dus $h(x) = 6{,}11\sin(2(x - 2{,}850)) = 6{,}11\sin(2x - 5{,}70)$.
Dus $b \approx 6{,}11$, $c = 2$ en $d \approx 5{,}70$.

44 **a** u_1 en u_2 hebben dezelfde frequentie, dus $c = 880\pi$.
Voer in $y_1 = 0{,}05\sin(880\pi x) + 0{,}1\sin(880\pi x - 0{,}4\pi)$.
De optie maximum geeft $x \approx 0{,}00088$ en $y \approx 0{,}12$.
De optie zero (TI) of ROOT (Casio) geeft $x \approx 0{,}000313$.
Dus $u_4 = 0{,}12\sin(880\pi(t - 0{,}000313)) = 0{,}12\sin(880\pi t - 0{,}87)$.

b $u_5 = 2 \cdot 0{,}05\sin(880\pi t) + 0{,}1\sin(880\pi t - 0{,}4\pi)$

$= 0{,}1\sin(880\pi t) + 0{,}1\sin(880\pi t - 0{,}4\pi)$

$= 0{,}1 \cdot 2\sin(\frac{1}{2}(880\pi t + 880\pi t - 0{,}4\pi)) \cdot \cos(\frac{1}{2}(880\pi t - 880\pi t + 0{,}4\pi))$

$= 0{,}2\sin(880\pi t - 0{,}2\pi) \cdot \cos(0{,}2\pi)$

$= 0{,}2\cos(0{,}2\pi)\sin(880\pi t - 0{,}2\pi)$

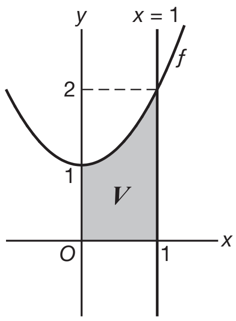
$\approx 0{,}16\sin(880\pi t - 0{,}2\pi)$

c De periodes van u_1, u_2 en u_3 zijn niet gelijk dus u is niet te schrijven in de vorm $u = b\sin(ct - d)$ dus de trilling is geen harmonische trilling maar vertoont wel zweving.

	$2u_1 = 0{,}1\sin(880\pi t)$	$u_2 = 0{,}1\sin(880\pi t - 0{,}4\pi)$	$u_3 = 0{,}2\sin(884\pi t)$
in $[0, 2\pi]$	880π periodes	880π periodes	884π periodes
in $[0, \frac{1}{2}]$	220 periodes	220 periodes	221 periodes

Dus de periode van de zweving is $\frac{1}{2}$ seconde.

45 **a**



$f(x) = x^2 + 1$ geeft $f'(x) = 2x$

omtrek $V = f(0) + 1 + f(1) + \int_0^1 \sqrt{1+(2x)^2}\,\mathrm{d}x = 1 + 1 + 2 + \int_0^1 \sqrt{1+4x^2}\,\mathrm{d}x$

De optie fnInt (TI) of $\int \mathrm{d}x$ (Casio) geeft omtrek $V \approx 5{,}48$.

b De optie fnInt (TI) of $\int \mathrm{d}x$ (Casio) geeft

$O(V) = \int_0^1 (x^2+1)\,\mathrm{d}x \approx 1{,}333$ en $\int_0^1 x(x^2+1)\,\mathrm{d}x = 0{,}75$, dus $x_Z \approx \dfrac{0{,}75}{1{,}333} \approx 0{,}56$.

$y = x^2 + 1$ geeft $\left.\begin{array}{r} x^2 = y - 1 \\ x \geq 0 \end{array}\right\} x = \sqrt{y-1}$

De optie fnInt (TI) of $\int \mathrm{d}x$ (Casio) geeft $\int_0^1 y\,\mathrm{d}y + \int_1^2 y(1-\sqrt{y-1})\,\mathrm{d}y \approx 0{,}933$, dus $y_Z \approx \dfrac{0{,}933}{1{,}333} \approx 0{,}70$.

Dus $Z(0{,}56;\ 0{,}70)$.

c $y = x^2 + 1 \xrightarrow{\text{translatie } (-1,\,0)} y = (x+1)^2 + 1$

$y = (x+1)^2 + 1$ geeft $\left.\begin{array}{r} (x+1)^2 = y - 1 \\ x + 1 \geq 0 \end{array}\right\} x + 1 = \sqrt{y-1}$

$x = -1 + \sqrt{y-1}$

$x^2 = (-1 + \sqrt{y-1})^2$

$x^2 = 1 - 2\sqrt{y-1} + y - 1$

$x^2 = y - 2\sqrt{y-1}$

$I(\text{omwentelingslichaam}) = I(\text{cilinder}) + \int_1^2 \pi x^2\,\mathrm{d}y$

$= \pi \cdot 1^2 \cdot 1 + \int_1^2 \pi(y - 2\sqrt{y-1})\,\mathrm{d}y$

$= \pi + \int_1^2 \pi(y - 2(y-1)^{\frac{1}{2}})\,\mathrm{d}y$

$= \pi + \left[\pi\left(\frac{1}{2}y^2 - \dfrac{2}{1\frac{1}{2}}(y-1)^{1\frac{1}{2}}\right)\right]_1^2$

$= \pi + [\pi(\frac{1}{2}y^2 - 1\frac{1}{3}(y-1)\sqrt{y-1})]_1^2$

$= \pi + \pi(2 - 1\frac{1}{3} \cdot 1 \cdot \sqrt{1}) - \pi(\frac{1}{2} - 1\frac{1}{3} \cdot 0\sqrt{0})$

$= \pi + \pi \cdot \frac{2}{3} - \pi \cdot \frac{1}{2}$

$= 1\frac{1}{6}\pi$

d $I(\text{omwentelingslichaam}) = \int_0^1 \pi(x^2+1)^2\,\mathrm{d}x = \int_0^1 \pi(x^4 + 2x^2 + 1)\,\mathrm{d}x$

$= [\pi(\frac{1}{5}x^5 + \frac{2}{3}x^3 + x)]_0^1 = \pi(\frac{1}{5} + \frac{2}{3} + 1) - \pi \cdot 0 = \frac{28}{15}\pi$

$\int_0^1 \pi x(x^2+1)^2\,\mathrm{d}x = \int_0^1 \pi(x^5 + 2x^3 + x)\,\mathrm{d}x = [\pi(\frac{1}{6}x^6 + \frac{1}{2}x^4 + \frac{1}{2}x^2)]_0^1$

$= \pi(\frac{1}{6} + \frac{1}{2} + \frac{1}{2}) - \pi \cdot 0 = 1\frac{1}{6}\pi$

Dus de x-coördinaat van het zwaartepunt is $\dfrac{1\frac{1}{6}\pi}{\frac{28}{15}\pi} = \frac{5}{8}$.

e $y = x^2 + 1$ geeft $x^2 = y - 1$

$$I(\text{omwentelingslichaam}) = \int_0^1 \pi \cdot 1\,\mathrm{d}y + \int_1^2 \pi(y-1)\,\mathrm{d}y = [\pi y]_0^1 + [\pi(\tfrac{1}{2}y^2 - y)]_1^2$$

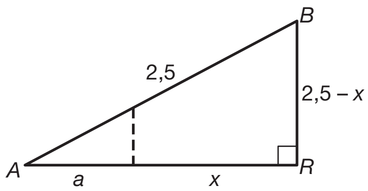
$$= \pi \cdot 1 - 0 + \pi(2-2) - \pi(\tfrac{1}{2} - 1) = 1\tfrac{1}{2}\pi$$

$$\int_0^1 \pi y\,\mathrm{d}y + \int_1^2 \pi y(y-1)\,\mathrm{d}y = \int_0^1 \pi y\,\mathrm{d}y + \int_1^2 \pi(y^2 - y)\,\mathrm{d}y = [\pi(\tfrac{1}{2}y^2)]_0^1 + [\pi(\tfrac{1}{3}y^3 - \tfrac{1}{2}y^2)]_1^2$$

$$= \pi \cdot \tfrac{1}{2} - 0 + \pi(\tfrac{8}{3} - 2) - \pi(\tfrac{1}{3} - \tfrac{1}{2}) = 1\tfrac{1}{3}\pi$$

Dus de y-coördinaat van het zwaartepunt is $\dfrac{1\frac{1}{3}\pi}{1\frac{1}{2}\pi} = \frac{8}{9}$.

46



$AQ + BR = 2{,}5$ dus $BR = 2{,}5 - AQ = 2{,}5 - x$

In $\triangle ARB$ is $AR^2 + BR^2 = AB^2$

$(a+x)^2 + (2{,}5 - x)^2 = 2{,}5^2$

$(a+x)^2 + 6{,}25 - 5x + x^2 = 6{,}25$

$(a+x)^2 = 5x - x^2$ dus $a + x = \sqrt{5x - x^2}$

Hieruit volgt $a = -x + \sqrt{5x - x^2}$.

$$\frac{\mathrm{d}a}{\mathrm{d}x} = -1 + \frac{1}{2\sqrt{5x - x^2}} \cdot (5 - 2x) = -1 + \frac{5 - 2x}{2\sqrt{5x - x^2}}$$

$\dfrac{\mathrm{d}a}{\mathrm{d}x} = 0$ geeft $-1 + \dfrac{5 - 2x}{2\sqrt{5x - x^2}} = 0$

$$\frac{5 - 2x}{2\sqrt{5x - x^2}} = 1$$

$2\sqrt{5x - x^2} = 5 - 2x$ kwadrateren geeft $4(5x - x^2) = 25 - 20x + 4x^2$

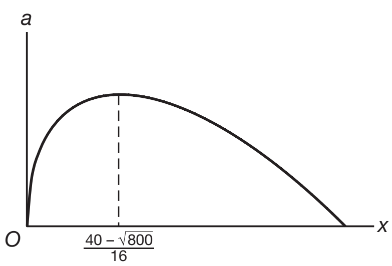
$20x - 4x^2 = 25 - 20x + 4x^2$

$8x^2 - 40x + 25 = 0$

$D = (-40)^2 - 4 \cdot 8 \cdot 25 = 800$

$x = \dfrac{40 - \sqrt{800}}{16} \vee x = \dfrac{40 + \sqrt{800}}{16}$

voldoet — voldoet niet

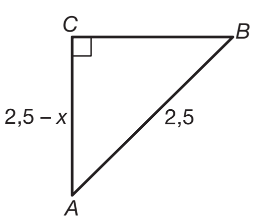


De onderkant van de garagedeur komt maximaal

$$-\frac{40 - \sqrt{800}}{16} + \sqrt{5 \cdot \frac{40 - \sqrt{800}}{16} - \left(\frac{40 - \sqrt{800}}{16}\right)^2} \approx 1{,}04 \text{ meter naar buiten.}$$

bladzijde 170

47 **a**



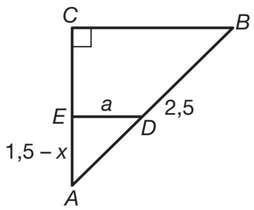
$x + AC = 2{,}5$ dus $AC = 2{,}5 - x$

In $\triangle ABC$ is $AC^2 + BC^2 = AB^2$

$(2{,}5 - x)^2 + BC^2 = 2{,}5^2$

$6{,}25 - 5x + x^2 + BC^2 = 6{,}25$

$BC^2 = 5x - x^2$ dus $BC = \sqrt{5x - x^2}$



$\triangle AED \backsim \triangle ACB$

$\frac{AE}{AC} = \frac{ED}{CB}$

$\frac{1{,}5 - x}{2{,}5 - x} = \frac{a}{\sqrt{5x - x^2}}$

$a(2{,}5 - x) = (1{,}5 - x)\sqrt{5x - x^2}$

$a = \frac{(1{,}5 - x)\sqrt{5x - x^2}}{2{,}5 - x}$

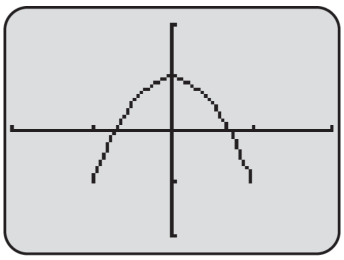
b Voer in $y_1 = \frac{(1{,}5 - x)\sqrt{5x - x^2}}{2{,}5 - x}$.

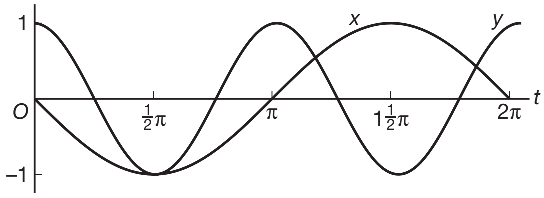
De optie maximum geeft $x \approx 0{,}66$ en $y \approx 0{,}77$.
Dus de maximale waarde van a is ongeveer 0,77 m.
De onderkant van de deur is dan ongeveer 66 cm omhoog geschoven.

c Het dak van een 1,5 m hoge auto moet minimaal 77 cm van de deur verwijderd zijn, om de garagedeur te kunnen openen of sluiten.

bladzijde 171

48 **a**





De grafieken van x en y hebben voor $t = \frac{1}{2}\pi$ en $t = 1\frac{1}{2}\pi$ een extreem en de grafieken van x en y hebben $t = \frac{1}{2}\pi$ en $t = 1\frac{1}{2}\pi$ als symmetrieas.
Dus de kleinste positieve waarden waarvoor de kromme precies één keer wordt doorlopen zijn $a = \frac{1}{2}\pi$ en $b = 1\frac{1}{2}\pi$.

b $y = x$ geeft $\cos(2t) = -\sin(t)$

$\sin(2t + \frac{1}{2}\pi) = \sin(t + \pi)$

$2t + \frac{1}{2}\pi = t + \pi + k \cdot 2\pi \ \vee \ 2t + \frac{1}{2}\pi = \pi - (t + \pi) + k \cdot 2\pi$

$t = \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ 2t + \frac{1}{2}\pi = \pi - t - \pi + k \cdot 2\pi$

$t = \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ 3t = -\frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ t = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

t op $[\frac{1}{2}\pi, 1\frac{1}{2}\pi]$ geeft $t = \frac{1}{2}\pi \ \vee \ t = 1\frac{1}{6}\pi$

$t = \frac{1}{2}\pi$ geeft $x = -\sin(\frac{1}{2}\pi) = -1$ en $y = x = -1$

$t = 1\frac{1}{6}\pi$ geeft $x = -\sin(1\frac{1}{6}\pi) = \frac{1}{2}$ en $y = x = \frac{1}{2}$

De snijpunten zijn $(-1, -1)$ en $(\frac{1}{2}, \frac{1}{2})$.

c K gaat door de punten $(-1, -1)$ en $(0, 1)$.

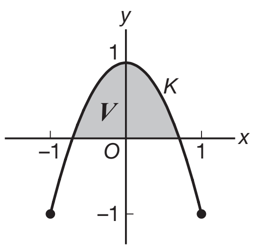
$\left.\begin{array}{l} y = ax^2 + b \\ \text{door } (0, 1) \end{array}\right\}$ $1 = a \cdot 0^2 + b$
$b = 1$ dus $y = ax^2 + 1$

$\left.\begin{array}{l} y = ax^2 + 1 \\ \text{door } (-1, -1) \end{array}\right\}$ $-1 = a \cdot (-1)^2 + 1$
$-1 = a + 1$
$a = -2$ dus $y = -2x^2 + 1$

Vermoedelijk hoort de formule $y = -2x^2 + 1$ bij K.
Substitutie van $x = -\sin(t)$ en $y = \cos(2t)$ in $y = -2x^2 + 1$ geeft
$\cos(2t) = -2(-\sin(t))^2 + 1$
$1 - 2\sin^2(t) = -2\sin^2(t) + 1$

$\left.\begin{array}{l} \text{Dit klopt voor elke } t. \\ x = -\sin(t) \\ -1 \le -\sin(t) \le 1 \end{array}\right\}$ Bij K hoort de formule $y = -2x^2 + 1$ met $-1 \le x \le 1$.

d



$y = 0$ geeft $-2x^2 + 1 = 0$

$-2x^2 = -1$

$x^2 = \frac{1}{2}$

$x = -\sqrt{\frac{1}{2}} \vee x = \sqrt{\frac{1}{2}}$

$y = -2x^2 + 1$ geeft $\frac{dy}{dx} = -4x$

omtrek $= \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \sqrt{1 + (-4x)^2}\, dx + \sqrt{\frac{1}{2}} - -\sqrt{\frac{1}{2}} = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \sqrt{1 + 16x^2}\, dx + 2\sqrt{\frac{1}{2}}$

De optie fnInt (TI) of $\int dx$ (Casio) geeft omtrek $V \approx 3{,}98$.

e $O(V) = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} (-2x^2 + 1)\, dx$

De optie fnInt (TI) of $\int dx$ (Casio) geeft $O(V) \approx 0{,}94$.

49 **a** $y = \frac{1}{2}\sqrt{3}$ geeft $\cos(2t) = \frac{1}{2}\sqrt{3}$

$2t = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \ 2t = -\frac{1}{6}\pi + k \cdot 2\pi$

$t = \frac{1}{12}\pi + k \cdot \pi \quad \vee \ t = -\frac{1}{12}\pi + k \cdot \pi$

t op $[\frac{1}{2}\pi, 1\frac{1}{2}\pi]$ geeft $t = 1\frac{1}{12}\pi \vee t = \frac{11}{12}\pi$

$t = \frac{11}{12}\pi$ geeft $x = \sin(3 \cdot \frac{11}{12}\pi) = \sin(2\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$, dus $A(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{3})$

$t = 1\frac{1}{12}\pi$ geeft $x = \sin(3 \cdot 1\frac{1}{12}\pi) = \sin(3\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$, dus $B(-\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{3})$

$AB = \frac{1}{2}\sqrt{2} - -\frac{1}{2}\sqrt{2} = \sqrt{2}$

b $t = \frac{1}{2}\pi + a$ geeft $x = \sin(3(\frac{1}{2}\pi + a)) = \sin(1\frac{1}{2}\pi + 3a) = \cos(1\frac{1}{2}\pi + 3a - \frac{1}{2}\pi) = \cos(3a + \pi) = -\cos(3a)$

en $\quad y = \cos(2(\frac{1}{2}\pi + a)) = \cos(\pi + 2a) = -\cos(2a)$

$t = 1\frac{1}{2}\pi - a$ geeft $x = \sin(3(1\frac{1}{2}\pi - a)) = \sin(4\frac{1}{2}\pi - 3a) = \cos(4\frac{1}{2}\pi - 3a - \frac{1}{2}\pi)$

$= \cos(4\pi - 3a) = \cos(-3a) = \cos(3a)$

en $\quad y = \cos(2(1\frac{1}{2}\pi - a)) = \cos(3\pi - 2a) = \cos(\pi - 2a) = -\cos(2a)$

Dus $S(-\cos(3a), -\cos(2a))$ en $T(\cos(3a), -\cos(2a))$.

Dit geeft $ST = |\cos(3a) - -\cos(3a)| = |2\cos(3a)|$.

50 **a** $x(t) = \cos(15t) + \cos(2t) = 2\cos(\frac{1}{2}(15t + 2t)) \cdot \cos(\frac{1}{2}(15t - 2t))$

$= 2\cos(8\frac{1}{2}t) \cdot \cos(6\frac{1}{2}t) = 2\cos(6\frac{1}{2}t)\cos(8\frac{1}{2}t)$

$y(t) = \sin(15t) + \sin(2t) = 2\sin(\frac{1}{2}(15t + 2t)) \cdot \cos(\frac{1}{2}(15t - 2t))$

$= 2\sin(8\frac{1}{2}t) \cdot \cos(6\frac{1}{2}t) = 2\cos(6\frac{1}{2}t)\sin(8\frac{1}{2}t)$

Dit geeft $\begin{cases} x(t) = r(t)\cos(8\frac{1}{2}t) \\ y(t) = r(t)\sin(8\frac{1}{2}t) \end{cases}$ met $r(t) = 2\cos(6\frac{1}{2}t)$.

b $x(t) = 0$ geeft $2\cos(6\frac{1}{2}t)\cos(8\frac{1}{2}t) = 0$

$\cos(6\frac{1}{2}t) = 0 \quad \vee \cos(8\frac{1}{2}t) = 0$

$6\frac{1}{2}t = \frac{1}{2}\pi + k \cdot \pi \ \vee \ 8\frac{1}{2}t = \frac{1}{2}\pi + k \cdot \pi$

$t = \frac{1}{13}\pi + k \cdot \frac{2}{13}\pi \quad \vee \ t = \frac{1}{17}\pi + k \cdot \frac{2}{17}\pi$

$y(t) = 0$ geeft $2\cos(6\frac{1}{2}t)\sin(8\frac{1}{2}t) = 0$

$\cos(6\frac{1}{2}t) = 0 \ \vee \ \sin(8\frac{1}{2}t) = 0$

$6\frac{1}{2}t = \frac{1}{2}\pi + k \cdot \pi \ \vee \ 8\frac{1}{2}t = k \cdot \pi$

$t = \frac{1}{13}\pi + k \cdot \frac{2}{13}\pi \ \vee \ t = k \cdot \frac{2}{17}\pi$

$$\left.\begin{array}{l}\text{Dus } x(t)=0 \wedge y(t)=0 \text{ voor } t=\frac{1}{13}\pi+k\cdot\frac{2}{13}\pi.\\ 0\le t\le 2\pi\end{array}\right\} \begin{array}{l} t=\frac{1}{13}\pi \vee t=\frac{3}{13}\pi \vee t=\frac{5}{13}\pi \vee t=\frac{7}{13}\pi \vee t=\frac{9}{13}\pi \\ \vee\ t=\frac{11}{13}\pi \vee t=\pi \vee t=1\frac{2}{13}\pi \vee t=1\frac{4}{13}\pi \\ \vee\ t=1\frac{6}{13}\pi \vee t=1\frac{8}{13}\pi \vee t=1\frac{10}{13}\pi \vee t=1\frac{12}{13}\pi\end{array}$$

Dus P passeert 13 keer het punt (0, 0).

bladzijde 172

51 **a** $y=0$ geeft $\sin(2t+\frac{1}{3}\pi)=0$

$$2t+\frac{1}{3}\pi=k\cdot\pi$$
$$2t=-\frac{1}{3}\pi+k\cdot\pi$$
$$t=-\frac{1}{6}\pi+k\cdot\frac{1}{2}\pi$$

Dit geeft de punten $(\sin(-\frac{1}{6}\pi), 0)$, $(\sin(\frac{1}{3}\pi), 0)$, $(\sin(\frac{5}{6}\pi), 0)$ en $(\sin(1\frac{1}{3}\pi), 0)$ ofwel $(-\frac{1}{2}, 0)$, $(\frac{1}{2}\sqrt{3}, 0)$, $(\frac{1}{2}, 0)$ en $(-\frac{1}{2}\sqrt{3}, 0)$.

b $AB=\sin(2a+\frac{1}{3}\pi)-\sin(2(\pi-a)+\frac{1}{3}\pi)$

$$=\sin(2a+\frac{1}{3}\pi)-\sin(2\frac{1}{3}\pi-2a)$$
$$=2\sin(\frac{1}{2}(2a+\frac{1}{3}\pi-2\frac{1}{3}\pi+2a))\cdot\cos(\frac{1}{2}(2a+\frac{1}{3}\pi+2\frac{1}{3}\pi-2a))$$
$$=2\sin(2a-\pi)\cdot\cos(1\frac{1}{3}\pi)$$
$$=2\sin(2a+\pi)\cdot-\frac{1}{2}$$
$$=-2\sin(2a)\cdot-\frac{1}{2}$$
$$=\sin(2a)$$

52 **a** $t_1=18$ geeft $z'(18)=100\cdot e^{0,1(18-40)}=100\cdot e^{-2,2}$

$z'(t)=100\cdot e^{-2,2}$ geeft $100\cdot e^{-0,2(t-100)}=100\cdot e^{-2,2}$

$$-0,2(t-100)=-2,2$$
$$t-100=11$$
$$t=111, \text{ dus } t_3=111$$

b $z(t)=a\cdot e^{0,1(t-40)}+b$ geeft $z'(t)=a\cdot 0,1\,e^{0,1(t-40)}=0,1a\,e^{0,1(t-40)}$

$z'(t)=100\cdot e^{0,1(t-40)}$ geeft $0,1a\,e^{0,1(t-40)}=100\,e^{0,1(t-40)}$ dus $0,1a=100$ ofwel $a=1000$.

Dus $z(t)=1000\,e^{0,1(t-40)}+b$.
$z(0)=30$ geeft $1000\cdot e^{0,1\cdot -40}+b=30$

$$1000\cdot e^{-4}+b=30$$
$$b=30-\frac{1000}{e^4}$$

Dus $a=1000$ en $b=30-\frac{1000}{e^4}$.

c $z(100)=z(0)+\int_0^{40} z'(s)\,ds+\int_{40}^{100} z'(s)\,ds=30+\int_0^{40} 100\cdot e^{0,1(s-40)}\,ds+\int_{40}^{100} 100\,ds$

De optie fnInt (TI) of $\int dx$ (Casio) geeft $z(100)\approx 7011,68$.

d $z(120)=z(100)+\int_{100}^{120} z'(s)\,ds\approx 7011,68+\int_{100}^{120} 100\cdot e^{-0,2(s-100)}\,ds$

De optie fnInt (TI) of $\int dx$ (Casio) geeft $z(120)\approx 7503$ kg.

bladzijde 173

53 **a** P haalt Q voor het eerst in als $\frac{11}{10}t=t+\frac{2}{3}\pi$

$$\frac{1}{10}t=\frac{2}{3}\pi$$
$$t=\frac{20}{3}\pi\approx 21$$

Dus na ongeveer 21 seconden haalt P voor het eerst Q in.

b $\dfrac{x_P + x_Q}{2} = \dfrac{5\cos(\frac{11}{10}t) + 5\cos(t + \frac{2}{3}\pi)}{2} = 2\frac{1}{2}(\cos(\frac{11}{10}t) + \cos(t + \frac{2}{3}\pi))$

$= 2\frac{1}{2} \cdot 2\cos(\frac{1}{2}(\frac{11}{10}t + t + \frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{11}{10}t - (t + \frac{2}{3}\pi)))$

$= 5\cos(\frac{1}{2}(\frac{21}{10}t + \frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{1}{10}t - \frac{2}{3}\pi))$

$= 5\cos(\frac{21}{20}t + \frac{1}{3}\pi)\cos(\frac{1}{20}t - \frac{1}{3}\pi))$

$\dfrac{y_P + y_Q}{2} = \dfrac{5\sin(\frac{11}{10}t) + 5\sin(t + \frac{2}{3}\pi)}{2} = 2\frac{1}{2}(\sin(\frac{11}{10}t) + \sin(t + \frac{2}{3}\pi))$

$= 2\frac{1}{2} \cdot 2\sin(\frac{1}{2}(\frac{11}{10}t + t + \frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{11}{10}t - (t + \frac{2}{3}\pi)))$

$= 5\sin(\frac{1}{2}(\frac{21}{10}t + \frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{1}{10}t - \frac{2}{3}\pi))$

$= 5\sin(\frac{21}{20}t + \frac{1}{3}\pi)\cos(\frac{1}{20}t - \frac{1}{3}\pi)$

Dus $\begin{cases} x(t) = 5\cos(\frac{21}{20}t + \frac{1}{3}\pi)\cos(\frac{1}{20}t - \frac{1}{3}\pi) \\ y(t) = 5\sin(\frac{21}{20}t + \frac{1}{3}\pi)\cos(\frac{1}{20}t - \frac{1}{3}\pi) \end{cases}$ ofwel $\begin{cases} x(t) = 5\cos(\frac{1}{20}t - \frac{1}{3}\pi)\cos(\frac{21}{10}t + \frac{1}{3}\pi) \\ y(t) = 5\cos(\frac{1}{20}t - \frac{1}{3}\pi)\sin(\frac{21}{20}t + \frac{1}{3}\pi) \end{cases}$

en hieruit volgt $\varphi(t) = 5\cos(\frac{1}{20}t - \frac{1}{3}\pi)$.

bladzijde 174

54 **a** $f(x) = \dfrac{1}{x} = x^{-1}$ geeft $f'(x) = -x^{-2} = -\dfrac{1}{x^2}$

Stel $y = ax + b$ met $a = f'(2) = -\dfrac{1}{2^2} = -\frac{1}{4}$.

$\left.\begin{array}{l} y = -\frac{1}{4}x + b \\ f(2) = \frac{1}{2} \end{array}\right\}$ $\frac{1}{2} = -\frac{1}{4} \cdot 2 + b$

$\frac{1}{2} = -\frac{1}{2} + b$

$1 = b$

$\left.\begin{array}{l} \text{Dus } y = -\frac{1}{4}x + 1 \\ A(4, 0) \end{array}\right\}$ $0 = -\frac{1}{4} \cdot 4 + 1$

$0 = -1 + 1$ klopt, dus de raaklijn in het punt met x-coördinaat 2 gaat door A.

b $f(x) = 4$ geeft $\dfrac{1}{x} = 4$

$x = \frac{1}{4}$, dus $Q(\frac{1}{4}, 4)$

$f(4) = \frac{1}{4}$, dus $P(4, \frac{1}{4})$

omtrek $V = 4 + \frac{1}{4} + \displaystyle\int_{\frac{1}{4}}^{4} \sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\,\mathrm{d}x + \frac{1}{4} + 4 = 8\frac{1}{2} + \int_{\frac{1}{4}}^{4} \sqrt{1 + \frac{1}{x^4}}\,\mathrm{d}x$

De optie fnInt (TI) of $\int \mathrm{d}x$ (Casio) geeft omtrek $V \approx 14{,}80$.

c $O(V) = \frac{1}{4} \cdot 4 + \displaystyle\int_{\frac{1}{4}}^{4} f(x)\,\mathrm{d}x = 1 + \int_{\frac{1}{4}}^{4} \frac{1}{x}\,\mathrm{d}x = 1 + [\ln|x|]_{\frac{1}{4}}^{4} = 1 + \ln(4) - \ln(\frac{1}{4})$

$= 1 + \ln(4) - \ln(4^{-1}) = 1 + \ln(4) + \ln(4) = 1 + 2\ln(4)$

d $\mathrm{rc}_{AC} = -1$, dus $f'(x) = -1$

$-\dfrac{1}{x^2} = -1$

$x^2 = 1$, dus $x = 1$

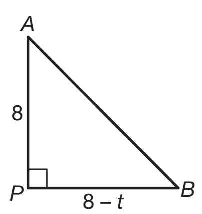
$\left.\begin{array}{l} y = -x + b \\ f(1) = 1 \end{array}\right\}$ $1 = -1 + b$

$2 = b$, dus $y = -x + 2$ is de formule van de raaklijn.

Het snijpunt van de raaklijn met de y-as is $(0, 2)$, dus $a = 2$.

bladzijde 175

55 **a**



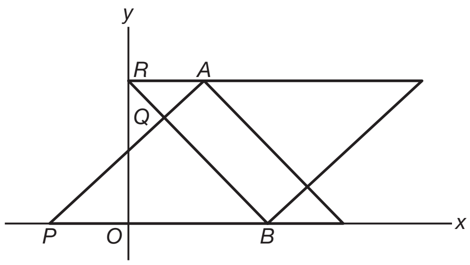
In $\triangle ABP$ is $AB^2 = AP^2 + BP^2$

$AB^2 = 8^2 + (8 - t)^2$

$AB^2 = 64 + 64 - 16t + t^2$

$AB^2 = 128 - 16t + t^2$, dus $a(t) = AB = \sqrt{128 - 16t + t^2}$

b



$\triangle BPQ$ is een gelijkbenige rechthoekige driehoek met $BP = 16 - t$, dus $BQ = \dfrac{16 - t}{\sqrt{2}}$.

$\triangle AQR$ is een gelijkbenige rechthoekige driehoek met $AR = t$, dus $AQ = \dfrac{t}{\sqrt{2}}$.

$G(t) = BQ \cdot AQ = \dfrac{16 - t}{\sqrt{2}} \cdot \dfrac{t}{\sqrt{2}} = \dfrac{16t - t^2}{2} = 8t - \frac{1}{2}t^2 = -\frac{1}{2}t^2 + 8t$

c $G(t) = -\frac{1}{2}t^2 + 8t$ geeft $G'(t) = -t + 8$

$G'(t) = 0$ geeft $-t + 8 = 0$, dus $t = 8$.

$a(t) = \sqrt{128 - 16t + t^2}$ geeft

$a'(t) = \dfrac{1}{2\sqrt{128 - 16t + t^2}} \cdot (-16t + 2t) = \dfrac{-8 + t}{\sqrt{128 - 16t + t^2}}$

$a'(t) = 0$ geeft $-8 + t = 0$

$t = 8$

Dus G en a bereiken beide op $t = 8$ hun uiterste waarde.

d $G = c - \frac{1}{2}a^2 = c - \frac{1}{2}(\sqrt{128 - 16t + t^2})^2 = c - \frac{1}{2}(128 - 16t + t^2) = c - 64 + 8t - \frac{1}{2}t^2$

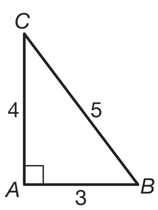
$G(t) = -\frac{1}{2}t^2 + 8t$

$\Big\}$ $c - 64 = 0$

$c = 64$

bladzijde 176

56 **a**



$O(ABC) = \frac{1}{2} \cdot 3 \cdot 4 = 6$

$s = \frac{1}{2}(3 + 4 + 5) = 6$

$H = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{36} = 6$

b $s = \frac{1}{2}(3 + 7 + x) = \frac{1}{2}(10 + x) = 5 + \frac{1}{2}x$

$H(x) = \sqrt{(5 + \frac{1}{2}x)(5 + \frac{1}{2}x - 3)(5 + \frac{1}{2}x - 7)(5 + \frac{1}{2}x - x)}$

$= \sqrt{(5 + \frac{1}{2}x)(2 + \frac{1}{2}x)(\frac{1}{2}x - 2)(5 - \frac{1}{2}x)}$

$= \sqrt{(5 + \frac{1}{2}x)(5 - \frac{1}{2}x)(\frac{1}{2}x + 2)(\frac{1}{2}x - 2)}$

$= \sqrt{(25 - \frac{1}{4}x^2)(\frac{1}{4}x^2 - 4)}$

c $H(x) = \sqrt{(25 - \frac{1}{4}x^2)(\frac{1}{4}x^2 - 4)} = \sqrt{-100 + \frac{29}{4}x^2 - \frac{1}{16}x^4}$

$H'(x) = \dfrac{1}{2\sqrt{-100 + \frac{29}{4}x^2 - \frac{1}{16}x^4}} \cdot (\frac{29}{2}x - \frac{1}{4}x^3) = \dfrac{\frac{29}{2}x - \frac{1}{4}x^3}{2\sqrt{-100 + \frac{29}{4}x^2 - \frac{1}{16}x^4}} = \dfrac{58x - x^3}{8\sqrt{-100 + \frac{29}{4}x^2 - \frac{1}{16}x^4}}$

$H'(x) = 0$ geeft $58x - x^3 = 0$

$x(58 - x^2) = 0$

$x = 0 \vee x^2 = 58$

$\left.\begin{array}{l} x = 0 \vee x = \sqrt{58} \vee x = -\sqrt{58} \\ 4 < x < 10 \end{array}\right\} x = \sqrt{58}$

bladzijde 177

57 **a** $l(t) = A'B' = x_A - x_B = \cos(t - \frac{1}{6}\pi) - \cos(t + \frac{1}{6}\pi)$

$= -2\sin(\frac{1}{2}(t - \frac{1}{6}\pi + t + \frac{1}{6}\pi))\sin(\frac{1}{2}(t - \frac{1}{6}\pi - (t + \frac{1}{6}\pi)))$

$= -2\sin(\frac{1}{2} \cdot 2t)\sin(\frac{1}{2}(t - \frac{1}{6}\pi - t - \frac{1}{6}\pi))$

$= -2\sin(t)\sin(\frac{1}{2} \cdot -\frac{1}{3}\pi)$

$= -2\sin(t)\sin(-\frac{1}{6}\pi)$

$= -2\sin(t) \cdot -\frac{1}{2}$

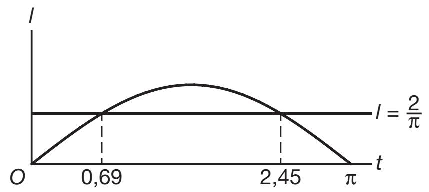
$= \sin(t)$

b $g = \frac{1}{\pi}\int_0^{\pi} l(t)\,dt = \frac{1}{\pi}\int_0^{\pi}\sin(t)\,dt = \frac{1}{\pi}[-\cos(t)]_0^{\pi} = \frac{1}{\pi}(-\cos(\pi) + \cos(0)) = \frac{1}{\pi}(1+1) = \frac{2}{\pi}$

c $l(t) = \frac{2}{\pi}$ geeft $\sin(t) = \frac{2}{\pi}$

Voer in $y_1 = \sin(x)$ en $y_2 = \frac{2}{\pi}$.

De optie intersect geeft $x \approx 0{,}69$ en $x \approx 2{,}45$.



$l(t) > \frac{2}{\pi}$ gedurende ongeveer $2{,}45 - 0{,}69 = 1{,}76$ seconden.

$l(t) < \frac{2}{\pi}$ gedurende ongeveer $\pi - 1{,}76 \approx 1{,}38$ seconden.

Dus de delen zijn niet even groot.