11 Goniometrie en beweging

25 **a** $\dfrac{2\sin(x)\cdot\cos(x)}{1-2\sin^2(x)} = \dfrac{\sin(2x)}{\cos(2x)} = \tan(2x)$

b $\cos^4(x) - \sin^4(x) = (\cos^2(x) + \sin^2(x))(\cos^2(x) - \sin^2(x)) = 1 \cdot \cos(2x) = \cos(2x)$

c $\dfrac{\sin(2x)}{1+\cos(2x)} = \dfrac{2\sin(x)\cos(x)}{1+2\cos^2(x)-1} = \dfrac{2\sin(x)\cos(x)}{2\cos^2(x)} = \dfrac{\sin(x)}{\cos(x)} = \tan(x)$

d $\cos(x-y)\cdot\cos(y) - \sin(x-y)\cdot\sin(y) = \cos(x-y+y) = \cos(x)$

26 **a** $\sin(x)\cdot\cos(x) = \frac{1}{4}$

$\frac{1}{2}\sin(2x) = \frac{1}{4}$

$\sin(2x) = \frac{1}{2}$

$2x = \frac{1}{6}\pi + k\cdot 2\pi \vee 2x = \frac{5}{6}\pi + k\cdot 2\pi$

$x = \frac{1}{12}\pi + k\cdot\pi \vee x = \frac{5}{12}\pi + k\cdot\pi$

b $\cos(x - \frac{1}{3}\pi) = \sin(2x)$

$\cos(x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{2}\pi)$

$x - \frac{1}{3}\pi = 2x - \frac{1}{2}\pi + k\cdot 2\pi \vee x - \frac{1}{3}\pi = -2x + \frac{1}{2}\pi + k\cdot 2\pi$

$-x = -\frac{1}{6}\pi + k\cdot 2\pi \vee 3x = \frac{5}{6}\pi + k\cdot 2\pi$

$x = \frac{1}{6}\pi + k\cdot 2\pi \vee x = \frac{5}{18}\pi + k\cdot\frac{2}{3}\pi$

c $\cos(x + \frac{1}{3}\pi) = -\sin(x)$

$\cos(x + \frac{1}{3}\pi) = \sin(x + \pi)$

$\cos(x + \frac{1}{3}\pi) = \cos(x + \frac{1}{2}\pi)$

$x + \frac{1}{3}\pi = x + \frac{1}{2}\pi + k\cdot 2\pi \vee x + \frac{1}{3}\pi = -x - \frac{1}{2}\pi + k\cdot 2\pi$

geen opl. $\quad 2x = -\frac{5}{6}\pi + k\cdot 2\pi$

$x = -\frac{5}{12}\pi + k\cdot\pi$

d $\cos(2x) - \sin^2(x) = \frac{1}{4}$

$1 - 2\sin^2(x) - \sin^2(x) = \frac{1}{4}$

$-3\sin^2(x) = -\frac{3}{4}$

$\sin^2(x) = \frac{1}{4}$

$\sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$

$x = \frac{1}{6}\pi + k\cdot 2\pi \vee x = \frac{5}{6}\pi + k\cdot 2\pi \vee x = -\frac{1}{6}\pi + k\cdot 2\pi \vee x = 1\frac{1}{6}\pi + k\cdot 2\pi$

$x = \frac{1}{6}\pi + k\cdot\pi \vee x = \frac{5}{6}\pi + k\cdot\pi$

27 **a** $f(x)=\frac{1}{2}$ geeft $\sin(x)=\frac{1}{2}$

$x=\frac{1}{6}\pi$

$f(x)=g(x)$ geeft $\sin(x)=\cos(x)$

$x=\frac{1}{4}\pi$

$g(x)=\frac{1}{2}$ geeft $\cos(x)=\frac{1}{2}$

$x=\frac{1}{3}\pi$

$$O(V)=\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}(\sin(x)-\tfrac{1}{2})\,dx+\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}(\cos(x)-\tfrac{1}{2})\,dx$$

$$=[-\cos(x)-\tfrac{1}{2}x]_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}+[\sin(x)-\tfrac{1}{2}x]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}$$

$$=-\cos(\tfrac{1}{4}\pi)-\tfrac{1}{8}\pi-(-\cos(\tfrac{1}{6}\pi)-\tfrac{1}{12}\pi)+\sin(\tfrac{1}{3}\pi)-\tfrac{1}{6}\pi-(\sin(\tfrac{1}{4}\pi)-\tfrac{1}{8}\pi)$$

$$=-\tfrac{1}{2}\sqrt{2}-\tfrac{1}{8}\pi+\tfrac{1}{2}\sqrt{3}+\tfrac{1}{12}\pi+\tfrac{1}{2}\sqrt{3}-\tfrac{1}{6}\pi-\tfrac{1}{2}\sqrt{2}+\tfrac{1}{8}\pi$$

$$=-\sqrt{2}+\sqrt{3}-\tfrac{1}{12}\pi$$

b $$I(L)=\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\pi\sin^2(x)\,dx+\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\pi\cos^2(x)\,dx-\pi\cdot(\tfrac{1}{2})^2\cdot(\tfrac{1}{3}\pi-\tfrac{1}{6}\pi)$$

$$=\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\pi(\tfrac{1}{2}-\tfrac{1}{2}\cos(2x))\,dx+\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\pi(\tfrac{1}{2}+\tfrac{1}{2}\cos(2x))\,dx-\tfrac{1}{24}\pi^2$$

$$=[\pi(\tfrac{1}{2}x-\tfrac{1}{4}\sin(2x))]_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}+[\pi(\tfrac{1}{2}x+\tfrac{1}{4}\sin(2x))]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}-\tfrac{1}{24}\pi^2$$

$$=\pi(\tfrac{1}{8}\pi-\tfrac{1}{4}\sin(\tfrac{1}{2}\pi))-\pi(\tfrac{1}{12}\pi-\tfrac{1}{4}\sin(\tfrac{1}{3}\pi))+\pi(\tfrac{1}{6}\pi+\tfrac{1}{4}\sin(\tfrac{2}{3}\pi))-\pi(\tfrac{1}{8}\pi+\tfrac{1}{4}\sin(\tfrac{1}{2}\pi))-\tfrac{1}{24}\pi^2$$

$$=\tfrac{1}{8}\pi^2-\tfrac{1}{4}\pi-\tfrac{1}{12}\pi^2+\tfrac{1}{8}\pi\sqrt{3}+\tfrac{1}{6}\pi^2+\tfrac{1}{8}\pi\sqrt{3}-\tfrac{1}{8}\pi^2-\tfrac{1}{4}\pi-\tfrac{1}{24}\pi^2$$

$$=-\tfrac{1}{2}\pi+\tfrac{1}{4}\pi\sqrt{3}+\tfrac{1}{24}\pi^2$$

c $$\text{omtrek}=\tfrac{1}{3}\pi-\tfrac{1}{6}\pi+\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\sqrt{1+(f'(x))^2}\,dx+\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\sqrt{1+(g'(x))^2}\,dx$$

$$=\tfrac{1}{6}\pi+\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\sqrt{1+\cos^2(x)}\,dx+\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\sqrt{1+\sin^2(x)}\,dx$$

De optie fnInt (TI) of $\int dx$ (Casio) geeft omtrek ≈ 1,19.

bladzijde 169

28 **a** $f(x)=2\cos^2(x)+\sin(2x)$ geeft $f'(x)=4\cos(x)\cdot -\sin(x)+2\cos(2x)=-2\sin(2x)+2\cos(2x)$

$f'(x)=0$ geeft $-2\sin(2x)+2\cos(2x)=0$

$2\cos(2x)=2\sin(2x)$

$\cos(2x)=\sin(2x)$

$\cos(2x)=\cos(2x-\frac{1}{2}\pi)$

$2x=2x-\frac{1}{2}\pi+k\cdot 2\pi \vee 2x=-2x+\frac{1}{2}\pi+k\cdot 2\pi$

geen opl. $\quad 4x=\frac{1}{2}\pi+k\cdot 2\pi$

$x=\frac{1}{8}\pi+k\cdot\frac{1}{2}\pi$

$f(x)=2\cos^2(x)+\sin(2x)$

$=2\cos^2(x)-1+1+\sin(2x)$

$=\cos(2x)+1+\sin(2x)$

max. is $f(\frac{1}{8}\pi)=\cos(\frac{1}{4}\pi)+1+\sin(\frac{1}{4}\pi)=\frac{1}{2}\sqrt{2}+1+\frac{1}{2}\sqrt{2}=1+\sqrt{2}$

min. is $f(\frac{5}{8}\pi)=\cos(\frac{5}{4}\pi)+1+\sin(\frac{5}{4}\pi)=-\frac{1}{2}\sqrt{2}+1-\frac{1}{2}\sqrt{2}=1-\sqrt{2}$

Dus $B_f=[1-\sqrt{2},1+\sqrt{2}]$.

b $f(x) = g(x)$ geeft $2\cos^2(x) + \sin(2x) = 2\sin(2x)$

$2\cos^2(x) = \sin(2x)$
$2\cos^2(x) - \sin(2x) = 0$
$2\cos^2(x) - 2\sin(x)\cos(x) = 0$
$2\cos(x)(\cos(x) - \sin(x)) = 0$
$\cos(x) = 0 \quad \vee \quad \cos(x) = \sin(x)$
$x = \frac{1}{2}\pi + k \cdot \pi \ \vee \ \cos(x) = \cos(x - \frac{1}{2}\pi)$
$x = \frac{1}{2}\pi + k \cdot \pi \ \vee \ x = x - \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ x = -x + \frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{1}{2}\pi + k \cdot \pi \ \vee$ geen opl. $\qquad 2x = \frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{1}{2}\pi + k \cdot \pi \ \vee \ x = \frac{1}{4}\pi + k \cdot \pi$
x op $[0, \pi]$ geeft $x = \frac{1}{4}\pi \ \vee \ x = \frac{1}{2}\pi$

$$O(V) = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2\sin(2x) - (2\cos^2(x) + \sin(2x))\,\mathrm{d}x$$
$$= \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2\sin(2x) - (2\cos^2(x) - 1 + 1 + \sin(2x))\,\mathrm{d}x$$
$$= \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (\sin(2x) - \cos(2x) - 1)\,\mathrm{d}x$$
$$= [-\tfrac{1}{2}\cos(2x) - \tfrac{1}{2}\sin(2x) - x]_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}$$
$$= -\tfrac{1}{2}\cos(\pi) - \tfrac{1}{2}\sin(\pi) - \tfrac{1}{2}\pi - (-\tfrac{1}{2}\cos(\tfrac{1}{2}\pi) - \tfrac{1}{2}\sin(\tfrac{1}{2}\pi) - \tfrac{1}{4}\pi)$$
$$= \tfrac{1}{2} - 0 - \tfrac{1}{2}\pi + 0 + \tfrac{1}{2} + \tfrac{1}{4}\pi = 1 - \tfrac{1}{4}\pi$$

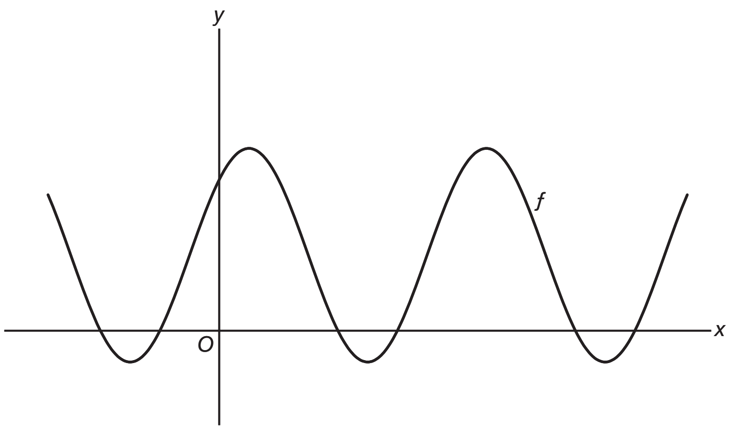
c C is het midden van AB als $g(p) = \frac{1}{2}f(p)$.

$2\sin(2p) = \cos^2(p) + \frac{1}{2}\sin(2p)$

$1\frac{1}{2}\sin(2p) = \cos^2(p)$

$3\sin(p)\cos(p) = \cos^2(p)$
$3\sin(p)\cos(p) - \cos^2(p) = 0$
$\cos(p)(3\sin(p) - \cos(p)) = 0$
$\cos(p) = 0 \quad \vee \ 3\sin(p) = \cos(p)$
vold. niet $\quad \vee \ 3\dfrac{\sin(p)}{\cos(p)} = 1$
$\tan(p) = \frac{1}{3}$

29 **a**



De grafiek van f is vermoedelijk lijnsymmetrisch in de verticale lijn door de eerste top rechts van de x-as.
$y = 2\cos^2(x) = 2u^2$ met $u = \cos(x)$
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4u \cdot -\sin(x) = 4\cos(x) \cdot -\sin(x) = -4\sin(x)\cos(x) = -2\sin(2x)$
$f'(x) = -2\sin(2x) + 2\cos(2x)$

$f'(x)=0$ geeft $-2\sin(2x)+2\cos(2x)=0$

$-2\sin(2x)=-2\cos(2x)$

$\sin(2x)=\cos(2x)$

$\cos(2x-\frac{1}{2}\pi)=\cos(2x)$

$2x-\frac{1}{2}\pi=2x+k\cdot 2\pi \vee 2x-\frac{1}{2}\pi=-2x+k\cdot 2\pi$

geen opl. $\quad 4x=\frac{1}{2}\pi+k\cdot 2\pi$

$x=\frac{1}{8}\pi+k\cdot\frac{1}{2}\pi$

Vermoedelijk is de grafiek van f lijnsymmetrisch in de lijn $x=\frac{1}{8}\pi$.

$$\begin{aligned}
f(\tfrac{1}{8}\pi+p) &= 2\cos^2(\tfrac{1}{8}\pi+p)+\sin(\tfrac{1}{4}\pi+2p)\\
&= 2\cos^2(\tfrac{1}{8}\pi+p)-1+1+\sin(\tfrac{1}{4}\pi+2p)\\
&= \cos(\tfrac{1}{4}\pi+2p)+\sin(\tfrac{1}{4}\pi+2p)+1\\
&= \cos(\tfrac{1}{4}\pi)\cos(2p)-\sin(\tfrac{1}{4}\pi)\sin(2p)+\sin(\tfrac{1}{4}\pi)\cos(2p)+\cos(\tfrac{1}{4}\pi)\sin(2p)+1\\
&= \tfrac{1}{2}\sqrt{2}\cos(2p)-\tfrac{1}{2}\sqrt{2}\sin(2p)+\tfrac{1}{2}\sqrt{2}\cos(2p)+\tfrac{1}{2}\sqrt{2}\sin(2p)+1\\
&= \sqrt{2}\cos(2p)+1\\
f(\tfrac{1}{8}\pi-p) &= 2\cos^2(\tfrac{1}{8}\pi-p)+\sin(\tfrac{1}{4}\pi-2p)\\
&= \cos(\tfrac{1}{4}\pi-2p)+\sin(\tfrac{1}{4}\pi-2p)+1\\
&= \cos(\tfrac{1}{4}\pi)\cos(2p)+\sin(\tfrac{1}{4}\pi)\sin(2p)+\sin(\tfrac{1}{4}\pi)\cos(2p)-\cos(\tfrac{1}{4}\pi)\sin(2p)+1\\
&= \tfrac{1}{2}\sqrt{2}\cos(2p)+\tfrac{1}{2}\sqrt{2}\sin(2p)+\tfrac{1}{2}\sqrt{2}\cos(2p)-\tfrac{1}{2}\sqrt{2}\sin(2p)+1\\
&= \sqrt{2}\cos(2p)+1
\end{aligned}$$

Dus voor elke p geldt $f(\frac{1}{8}\pi+p)=f(\frac{1}{8}\pi-p)$.
Hieruit volgt dat de grafiek van f puntsymmetrisch is in de lijn $x=\frac{1}{8}\pi$.

b $f(x)=2\cos^2(x)+\sin(2x)=2\cos^2(x)-1+1+\sin(2x)=\cos(2x)+1+\sin(2x)$

De evenwichtsstand is 1.

$f(x)=1$ geeft $\cos(2x)+1+\sin(2x)=1$

$\cos(2x)=-\sin(2x)$

$\sin(2x+\frac{1}{2}\pi)=\sin(-2x)$

$2x+\frac{1}{2}\pi=-2x+k\cdot 2\pi \vee 2x+\frac{1}{2}\pi=\pi+2x+k\cdot 2\pi$

$4x=-\frac{1}{2}\pi+k\cdot 2\pi$ $\quad$ geen oplossing

$x=-\frac{1}{8}\pi+k\cdot\frac{1}{2}\pi$

Vermoedelijk is de grafiek van f symmetrisch in het punt $A(-\frac{1}{8}\pi, 1)$.

$$\begin{aligned}
f(-\tfrac{1}{8}\pi-p) &= \cos(-\tfrac{1}{4}\pi-2p)+1+\sin(-\tfrac{1}{4}\pi-2p)\\
&= \cos(-\tfrac{1}{4}\pi)\cos(2p)+\sin(-\tfrac{1}{4}\pi)\sin(2p)+1+\sin(-\tfrac{1}{4}\pi)\cos(2p)-\cos(-\tfrac{1}{4}\pi)\sin(2p)\\
&= \tfrac{1}{2}\sqrt{2}\cos(2p)-\tfrac{1}{2}\sqrt{2}\sin(2p)+1-\tfrac{1}{2}\sqrt{2}\cos(2p)-\tfrac{1}{2}\sqrt{2}\sin(2p)\\
&= -\sqrt{2}\sin(2p)+1
\end{aligned}$$

$$\begin{aligned}
f(-\tfrac{1}{8}\pi+p) &= \cos(-\tfrac{1}{4}\pi+2p)+1+\sin(-\tfrac{1}{4}\pi+2p)\\
&= \cos(-\tfrac{1}{4}\pi)\cos(2p)-\sin(-\tfrac{1}{4}\pi)\sin(2p)+1+\sin(-\tfrac{1}{4}\pi)\cos(2p)+\cos(-\tfrac{1}{4}\pi)\sin(2p)\\
&= \tfrac{1}{2}\sqrt{2}\cos(2p)+\tfrac{1}{2}\sqrt{2}\sin(2p)+1-\tfrac{1}{2}\sqrt{2}\cos(2p)+\tfrac{1}{2}\sqrt{2}\sin(2p)\\
&= \sqrt{2}\sin(2p)+1
\end{aligned}$$

$f(-\frac{1}{8}\pi-p)+f(-\frac{1}{8}\pi+p)=-\sqrt{2}\sin(2p)+1+\sqrt{2}\sin(2p)+1=2$

Dus de grafiek van f is symmetrisch in het punt $A(-\frac{1}{8}\pi, 1)$.

30 **a** $f(x)=0$ geeft $2\sin^2(x)+\sin(x)=0$

$\sin(x)(2\sin(x)+1)=0$

$\sin(x)=0 \vee \sin(x)=-\frac{1}{2}$

$x=k\cdot\pi \vee x=-\frac{1}{6}\pi+k\cdot 2\pi \vee x=1\frac{1}{6}\pi+k\cdot 2\pi$

x op $[0, 2\pi]$ geeft de nulpunten 0, π, 2π, $1\frac{1}{6}\pi$ en $1\frac{5}{6}\pi$.

b $O = \int_0^{\pi} f(x)\,dx = \int_0^{\pi} (2\sin^2(x) + \sin(x))\,dx$

$= \int_0^{\pi} (2\sin^2(x) - 1 + \sin(x) + 1)\,dx = \int_0^{\pi} (-\cos(2x) + \sin(x) + 1)\,dx$

$= [-\frac{1}{2}\sin(2x) - \cos(x) + x]_0^{\pi}$

$= -\frac{1}{2}\sin(2\pi) - \cos(\pi) + \pi - (-\frac{1}{2}\sin(0) - \cos(0) + 0)$

$= 0 - -1 + \pi + 0 + 1 = 2 + \pi$

c $f(x) = 2\sin^2(x) + \sin(x)$ geeft $f'(x) = 4\sin(x)\cos(x) + \cos(x)$

$f'(x) = 0$ geeft $4\sin(x)\cos(x) + \cos(x) = 0$

$\cos(x)(4\sin(x) + 1) = 0$

$\cos(x) = 0 \quad \vee \quad \sin(x) = -\frac{1}{4}$

$x = \frac{1}{2}\pi \ \vee\ x = 1\frac{1}{2}\pi \ \vee\ \sin(x) = -\frac{1}{4}$

$x = \frac{1}{2}\pi$ geeft $f(x) = f(\frac{1}{2}\pi) = 2\sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) = 2 \cdot 1 + 1 = 3$

$x = 1\frac{1}{2}\pi$ geeft $f(x) = f(1\frac{1}{2}\pi) = 2\sin^2(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) = 2 \cdot 1 - 1 = 1$

$\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} = -\frac{1}{8}$

$f(x) = p$ heeft precies vier oplossingen voor $-\frac{1}{8} < p < 0 \ \vee\ 0 < p < 1$.

d De optie fnInt (TI) of $\int dx$ (Casio) geeft lengte $= \int_0^{2\pi} \sqrt{1 + (4\sin(x)\cos(x) + \cos(x))^2}\,dx \approx 11{,}07$.

31 **a** $g(x) = 2\sin(x) + \cos(2x)$ geeft

$g'(x) = 2\cos(x) - 2\sin(2x) = 2\cos(x) - 4\sin(x)\cos(x) = 2\cos(x)(1 - 2\sin(x))$

$g'(x) = 0$ geeft $\cos(x) = 0 \quad \vee\ \sin(x) = \frac{1}{2}$

$x = \frac{1}{2}\pi + k \cdot \pi \ \vee\ x = \frac{1}{6}\pi + k \cdot 2\pi \ \vee\ x = \frac{5}{6}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \ \vee\ x = 1\frac{1}{2}\pi \ \vee\ x = \frac{1}{6}\pi \ \vee\ x = \frac{5}{6}\pi$

Dit geeft de toppen $(\frac{1}{2}\pi, 1)$, $(1\frac{1}{2}\pi, -3)$, $(\frac{1}{6}\pi, 1\frac{1}{2})$ en $(\frac{5}{6}\pi, 1\frac{1}{2})$.

$f(\frac{1}{6}\pi) = 4 \cdot (\frac{1}{2}\sqrt{3})^2 - 3 \cdot \frac{1}{2} = 3 - 1\frac{1}{2} = 1\frac{1}{2}$

$f(\frac{5}{6}\pi) = 4 \cdot (-\frac{1}{2}\sqrt{3})^2 - 3 \cdot \frac{1}{2} = 3 - 1\frac{1}{2} = 1\frac{1}{2}$

Dus de toppen $(\frac{1}{6}\pi, 1\frac{1}{2})$ en $(\frac{5}{6}\pi, 1\frac{1}{2})$ liggen op de grafiek van f.

De toppen $(\frac{1}{2}\pi, 1)$ en $(1\frac{1}{2}\pi, -3)$ liggen niet op de grafiek van f, want $f(\frac{1}{2}\pi) = -3$ en $f(1\frac{1}{2}\pi) = 3$.
Dus er liggen twee toppen van de grafiek van g op de grafiek van f.

b $f(x) = g(x)$ geeft $4\cos^2(x) - 3\sin(x) = 2\sin(x) + \cos(2x)$

$4(1 - \sin^2(x)) - 5\sin(x) = 1 - 2\sin^2(x)$

$4 - 4\sin^2(x) - 5\sin(x) - 1 + 2\sin^2(x) = 0$

$-2\sin^2(x) - 5\sin(x) + 3 = 0$

Stel $\sin(x) = p$.

$-2p^2 - 5p + 3 = 0$

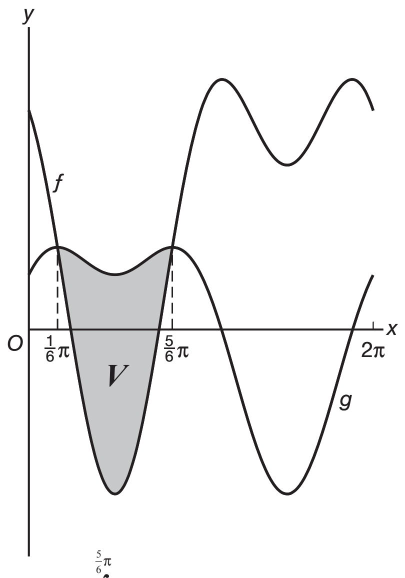
$D = 25 - 4 \cdot -2 \cdot 3 = 49$, dus $\sqrt{D} = 7$

$p = \frac{5 - 7}{-4} = \frac{1}{2} \ \vee\ p = \frac{5 + 7}{-4} = -3$

$\sin(x) = \frac{1}{2} \quad \vee \quad \sin(x) = -3$

$x = \frac{1}{6}\pi + k \cdot 2\pi \ \vee\ x = \frac{5}{6}\pi + k \cdot 2\pi \quad$ geen opl.

x op $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \ \vee\ x = \frac{5}{6}\pi$



$$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (g(x) - f(x))\,dx$$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin(x) + \cos(2x) - 4\cos^2(x) + 3\sin(x))\,dx$$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (5\sin(x) + \cos(2x) - 4(\tfrac{1}{2} + \tfrac{1}{2}\cos(2x))\,dx$$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (5\sin(x) + \cos(2x) - 2 - 2\cos(2x))\,dx$$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (5\sin(x) - \cos(2x) - 2)\,dx$$

$$= [-5\cos(x) - \tfrac{1}{2}\sin(2x) - 2x]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$$

$$= -5\cos(\tfrac{5}{6}\pi) - \tfrac{1}{2}\sin(1\tfrac{2}{3}\pi) - 1\tfrac{2}{3}\pi - (-5\cos(\tfrac{1}{6}\pi) - \tfrac{1}{2}\sin(\tfrac{1}{3}\pi) - \tfrac{1}{3}\pi)$$

$$= -5 \cdot -\tfrac{1}{2}\sqrt{3} - \tfrac{1}{2} \cdot -\tfrac{1}{2}\sqrt{3} - 1\tfrac{2}{3}\pi + 5 \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2} \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{3}\pi$$

$$= 5\tfrac{1}{2}\sqrt{3} - 1\tfrac{1}{3}\pi$$

c omtrek $= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \sqrt{1 + (f'(x))^2}\,dx + \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \sqrt{1 + (g'(x))^2}\,dx$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \sqrt{1 + (-8\cos(x)\sin(x) - 3\cos(x))^2}\,dx + \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \sqrt{1 + (2\cos(x) - 2\sin(2x))^2}\,dx$$

De optie fnInt (TI) of $\int dx$ (Casio) geeft omtrek ≈ 11,72.

bladzijde 170

32 **a** $f_p(x) = \sin^2(x) + p\cos(2x) = \sin^2(x) + p(1 - 2\sin^2(x))$
$= \sin^2(x) + p - 2p\sin^2(x) = (1 - 2p)\sin^2(x) + p$

De grafiek is een horizontale lijn als $1 - 2p = 0$, ofwel $p = \frac{1}{2}$.

b $f_p'(x) = 2\sin(x)\cos(x) - 2p\sin(2x)$
$= \sin(2x) - 2p\sin(2x)$
$= (1 - 2p)\sin(2x)$

$f_p'(x) = 1$ heeft geen oplossingen als $-1 < 1 - 2p < 1$
$-2 < -2p < 0$
$1 > p > 0$
$0 < p < 1$

Er zijn geen raaklijnen evenwijdig met $y = x$ voor $0 < p < 1$.

c $\int_0^a f_p(x)\,dx = \int_0^a (\sin^2(x) + p\cos(2x))\,dx = \int_0^a (\frac{1}{2} - \frac{1}{2}\cos(2x) + p\cos(2x))\,dx$

$= \int_0^a (\frac{1}{2} + (p - \frac{1}{2})\cos(2x))\,dx = [\frac{1}{2}x + \frac{1}{2}(p - \frac{1}{2})\sin(2x)]_0^a$

$= \frac{1}{2}a + \frac{1}{2}(p - \frac{1}{2})\sin(2a) - (0 + 0) = \frac{1}{2}a + \frac{1}{2}(p - \frac{1}{2})\sin(2a)$

Deze uitkomst is onafhankelijk van p als $\sin(2a) = 0$

$2a = k \cdot \pi$

$a = k \cdot \frac{1}{2}\pi$

a op $[0, \pi]$ geeft $a = 0 \lor a = \frac{1}{2}\pi \lor a = \pi$

33 **a** $f(x) = \dfrac{3\cos(x)}{2 + \sin(x)}$ geeft

$f'(x) = \dfrac{(2 + \sin(x)) \cdot -3\sin(x) - 3\cos(x) \cdot \cos(x)}{(2 + \sin(x))^2}$

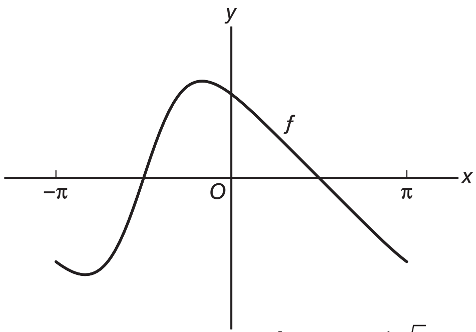
$= \dfrac{-6\sin(x) - 3\sin^2(x) - 3\cos^2(x)}{(2 + \sin(x))^2} = \dfrac{-6\sin(x) - 3(\sin^2(x) + \cos^2(x))}{(2 + \sin(x))^2} = \dfrac{-6\sin(x) - 3}{(2 + \sin(x))^2}$

$f'(x) = 0$ geeft $-6\sin(x) - 3 = 0$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \lor x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x op $[-\pi, \pi]$ geeft $x = -\frac{1}{6}\pi \lor x = -\frac{5}{6}\pi$



min. is $f(-\frac{5}{6}\pi) = \dfrac{3 \cdot \cos(-\frac{5}{6}\pi)}{2 - \frac{1}{2}} = \dfrac{3 \cdot -\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = -\sqrt{3}$

max. is $f(-\frac{1}{6}\pi) = \dfrac{3 \cdot \cos(-\frac{1}{6}\pi)}{2 - \frac{1}{2}} = \dfrac{3 \cdot \frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = \sqrt{3}$

Dus $B_f = [-\sqrt{3}, \sqrt{3}]$.

b $f(x) \cdot f(-x) = \frac{9}{7}$ geeft

$\dfrac{3\cos(x)}{2 + \sin(x)} \cdot \dfrac{3\cos(-x)}{2 + \sin(-x)} = \dfrac{9}{7}$

$\dfrac{3\cos(x) \cdot 3\cos(x)}{(2 + \sin(x))(2 - \sin(x))} = \dfrac{9}{7}$

$\dfrac{9\cos^2(x)}{4 - \sin^2(x)} = \dfrac{9}{7}$

$\dfrac{\cos^2(x)}{4 - \sin^2(x)} = \dfrac{1}{7}$

$7\cos^2(x) = 4 - \sin^2(x)$

$7\cos^2(x) = 4 - (1 - \cos^2(x))$

$7\cos^2(x) = 4 - 1 + \cos^2(x)$

$6\cos^2(x) = 3$

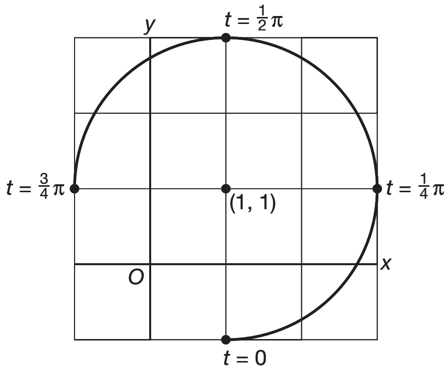
$\cos^2(x) = \frac{1}{2}$

$\cos(x) = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \lor \cos(x) = -\frac{1}{2}\sqrt{2}$

$x = \frac{1}{4}\pi + k \cdot 2\pi \lor x = -\frac{1}{4}\pi + k \cdot 2\pi \lor x = 1\frac{1}{4}\pi + k \cdot 2\pi \lor x = -1\frac{1}{4}\pi + k \cdot 2\pi$

x op $[-\pi, \pi]$ geeft $x = \frac{1}{4}\pi \lor x = -\frac{1}{4}\pi \lor x = -\frac{3}{4}\pi \lor x = \frac{3}{4}\pi$

34 **a** De baan van P is driekwartcirkel met middelpunt (1, 1) en straal 2.



b Substitutie van $x = 1 + 2\cos(2t - \frac{1}{2}\pi)$ en $y = 1 + 2\sin(2t - \frac{1}{2}\pi)$ in $y = -x + 2$ geeft

$1 + 2\sin(2t - \frac{1}{2}\pi) = -1 - 2\cos(2t - \frac{1}{2}\pi) + 2$

$2\sin(2t - \frac{1}{2}\pi) = -2\cos(2t - \frac{1}{2}\pi)$

$\sin(2t - \frac{1}{2}\pi) = -\cos(2t - \frac{1}{2}\pi)$

$\cos(2t - \pi) = \cos(2t + \frac{1}{2}\pi)$

$2t - \pi = 2t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \pi = -2t - \frac{1}{2}\pi + k \cdot 2\pi$

geen opl. $\quad 4t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$

t op $[0, \frac{3}{4}\pi]$ geeft $t = \frac{1}{8}\pi \vee t = \frac{5}{8}\pi$

$t = \frac{1}{8}\pi$ geeft $x_P = 1 + 2\cos(-\frac{1}{4}\pi) = 1 + 2 \cdot \frac{1}{2}\sqrt{2} = 1 + \sqrt{2}$

en $y_P = 1 + 2\sin(-\frac{1}{4}\pi) = 1 + 2 \cdot -\frac{1}{2}\sqrt{2} = 1 - \sqrt{2}$

$t = \frac{5}{8}\pi$ geeft $x_P = 1 + 2\cos(\frac{3}{4}\pi) = 1 - \sqrt{2}$

en $y_P = 1 + 2\sin(\frac{3}{4}\pi) = 1 + \sqrt{2}$

De snijpunten zijn $(1 + \sqrt{2}, 1 - \sqrt{2})$ en $(1 - \sqrt{2}, 1 + \sqrt{2})$.

c $x = 0$ geeft $1 + 2\cos(2t - \frac{1}{2}\pi) = 0$

$\cos(2t - \frac{1}{2}\pi) = -\frac{1}{2}$

$2t - \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee 2t - \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$2t = \frac{7}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{6}\pi + k \cdot 2\pi$

$t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{1}{12}\pi + k \cdot \pi$

$y = 0$ geeft $1 + 2\sin(2t - \frac{1}{2}\pi) = 0$

$\sin(2t - \frac{1}{2}\pi) = -\frac{1}{2}$

$2t - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t - \frac{1}{2}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$

$2t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2t = 1\frac{2}{3}\pi + k \cdot 2\pi$

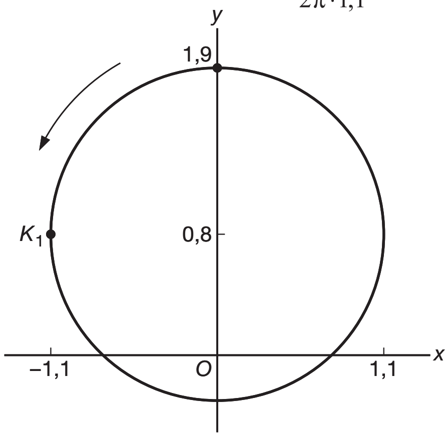
$t = \frac{1}{6}\pi + k \cdot \pi \vee t = \frac{5}{6}\pi + k \cdot \pi$

t op $[0, \frac{3}{4}\pi]$ geeft snijpunt met de x-as voor $t = \frac{1}{6}\pi$ en snijpunt met de y-as voor $t = \frac{7}{12}\pi$.

Dus beide coördinaten van P zijn positief voor $\frac{1}{6}\pi < t < \frac{7}{12}\pi$.

35 **a** $v = 0{,}5$ m/s, dus per seconde $\frac{0{,}5}{2\pi \cdot 1{,}1}$ gedeelte van een cirkel, dus $\frac{0{,}5}{2\pi \cdot 1{,}1} \cdot 2\pi$ rad/s $= \frac{5}{11}$ rad/s.

b



De omlooptijd is $\frac{2\pi}{\frac{5}{11}} = 4{,}4\pi$ seconde, dus op $t = 2{,}2\pi$ seconde bevindt de voorste koker (K_1) zich in (1,1; 0,8).

Dus $\begin{cases} x = 1{,}1\cos(\frac{5}{11}(t - 2{,}2\pi)) \\ y = 0{,}8 + 1{,}1\sin(\frac{5}{11}(t - 2{,}2\pi)) \end{cases}$ met t in seconden en x en y in meters.

c De volgende koker (K_2) loopt $\frac{1}{8}$ cirkel achter op K_1, dus K_2 bevindt zich op $t = 2{,}2\pi + \frac{1}{8} \cdot 4{,}4\pi = 2{,}75\pi$ seconde in (1,1; 0,8).

Dus $\begin{cases} x = 1{,}1\cos(\frac{5}{11}(t - 2{,}75\pi)) \\ y = 0{,}8 + 1{,}1\sin(\frac{5}{11}(t - 2{,}75\pi)) \end{cases}$ met t in seconden en x en y in meters.

d Los op $0{,}8 + 1{,}1\sin(\frac{5}{11}t) \leq 0$.

Voer in $y_1 = 0{,}8 + 1{,}1\sin(\frac{5}{11}x)$.
De optie zero (TI) of ROOT (Casio) geeft $x \approx 8{,}70$ en $x \approx 12{,}03$.
Dus een koker blijft per omwenteling ongeveer $12{,}03 - 8{,}70 \approx 3{,}3$ seconde onder water.

bladzijde 171

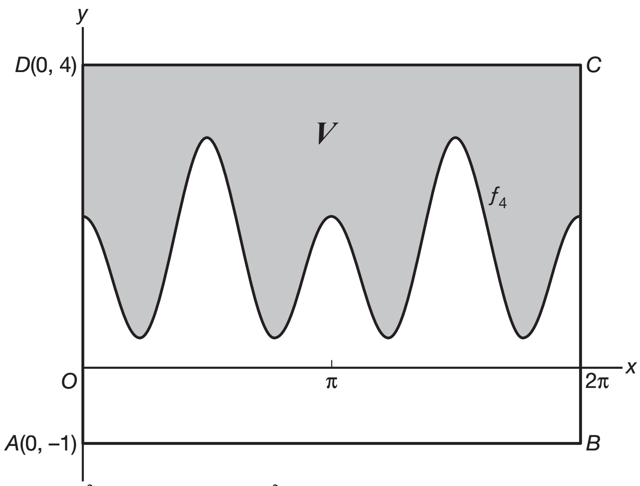
36 **a**
$$\begin{aligned} f_2(x) &= 1 + \sin^2(x) + \cos(2x) \\ &= 1 + \tfrac{1}{2} - \tfrac{1}{2}\cos(2x) + \cos(2x) \\ &= 1\tfrac{1}{2} + \tfrac{1}{2}\cos(2x) \\ &= 1\tfrac{1}{2} + \tfrac{1}{2}\sin(2x + \tfrac{1}{2}\pi) \\ &= 1\tfrac{1}{2} + \tfrac{1}{2}\sin(2(x + \tfrac{1}{4}\pi)) \end{aligned}$$

Dit geeft $a = 1\frac{1}{2}$, $b = \frac{1}{2}$, $c = 2$ en $d = -\frac{1}{4}\pi$.

b $f_n(\frac{1}{6}\pi) = \frac{1}{4}$ geeft $1 + \sin^2(\frac{1}{6}\pi) + \cos(n \cdot \frac{1}{6}\pi) = \frac{1}{4}$
$$1 + \tfrac{1}{4} + \cos(\tfrac{1}{6}\pi n) = \tfrac{1}{4}$$
$$\cos(\tfrac{1}{6}\pi n) = -1$$
$$\tfrac{1}{6}\pi n = \pi + k \cdot 2\pi$$
$$n = 6 + k \cdot 12$$
n tussen 0 en 50 geeft $n = 6 \lor n = 18 \lor n = 30 \lor n = 42$.

c
$$\begin{aligned} f_4(x) &= 1 + \sin^2(x) + \cos(4x) \\ &= 1 + \tfrac{1}{2} - \tfrac{1}{2}\cos(2x) + \cos(4x) \\ &= 1\tfrac{1}{2} - \tfrac{1}{2}\cos(2x) + \cos(4x) \end{aligned}$$

d



$$O(V) = \int_0^{2\pi} (4 - f_4(x))\,dx = \int_0^{2\pi} (4 - 1\tfrac{1}{2} + \tfrac{1}{2}\cos(2x) - \cos(4x))\,dx$$

$$= [2\tfrac{1}{2}x + \tfrac{1}{4}\sin(2x) - \tfrac{1}{4}\sin(4x)]_0^{2\pi}$$

$$= 5\pi + \tfrac{1}{4}\sin(4\pi) - \tfrac{1}{4}\sin(8\pi) - (0 + \tfrac{1}{4}\sin(0) - \tfrac{1}{4}\sin(0))$$

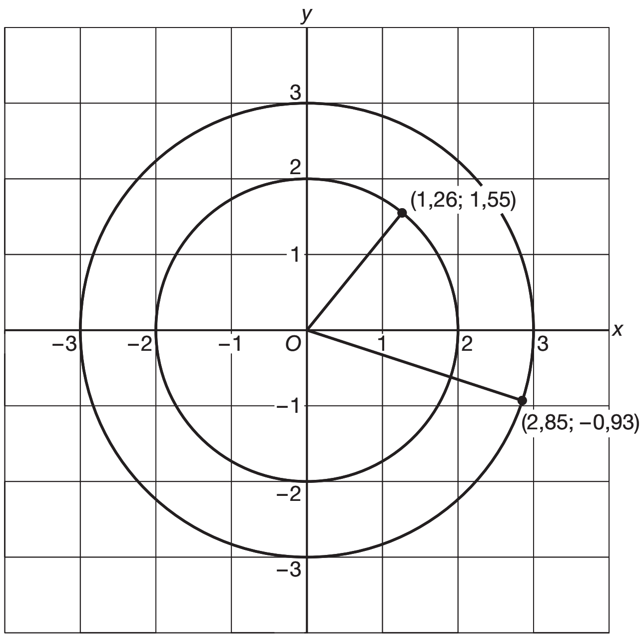
$$= 5\pi + 0 - 0 = 5\pi$$

$O(\text{rechthoek}) = 2\pi \cdot 5 = 10\pi$

Dus de grafiek van f_4 verdeelt de rechthoek in twee gebieden met dezelfde oppervlakte.

37 **a** $x = 3\sin(2\pi t) \wedge y = 3\cos(2\pi t)$ geeft de cirkel met middelpunt (0, 0) en straal 3.

$x = 2\sin(\tfrac{1}{6}\pi t) \wedge y = 2\cos(\tfrac{1}{6}\pi t)$ geeft de cirkel met middelpunt (0, 0) en straal 2.
grote wijzer: $t = 1{,}3$ geeft $x \approx 2{,}85 \wedge y \approx -0{,}93$
kleine wijzer: $t = 1{,}3$ geeft $x \approx 1{,}26 \wedge y \approx 1{,}55$



b Wijzers vallen over elkaar geeft
$\sin(2\pi t) = \sin(\tfrac{1}{6}\pi t) \wedge \cos(2\pi t) = \cos(\tfrac{1}{6}\pi t)$

$(2\pi t = \tfrac{1}{6}\pi t + k \cdot 2\pi \vee 2\pi t = \pi - \tfrac{1}{6}\pi t + k \cdot 2\pi) \wedge (2\pi t = \tfrac{1}{6}\pi t + k \cdot 2\pi \vee 2\pi t = -\tfrac{1}{6}\pi t + k \cdot 2\pi)$

$(\tfrac{11}{6}t = k \cdot 2 \vee \tfrac{13}{6}t = 1 + k \cdot 2) \wedge (\tfrac{11}{6}t = k \cdot 2 \vee \tfrac{13}{6}t = k \cdot 2)$

$(t = k \cdot \tfrac{12}{11} \vee t = \tfrac{6}{13} + k \cdot \tfrac{12}{13}) \wedge (t = k \cdot \tfrac{12}{11} \vee t = k \cdot \tfrac{12}{13})$

$t = k \cdot \tfrac{12}{11}$

Het eerste tijdstip na $t = 0$ is dus $t = \tfrac{12}{11}$.

c afstand $= \sqrt{(3\sin(2\pi t) - 2\sin(\frac{1}{6}\pi t))^2 + (3\cos(2\pi t) - 2\cos(\frac{1}{6}\pi t))^2}$

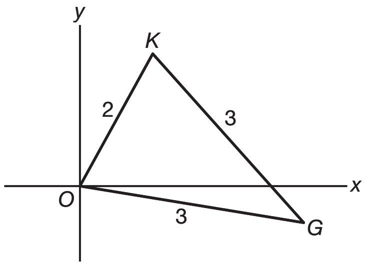
$= \sqrt{9\sin^2(2\pi t) - 12\sin(2\pi t)\sin(\frac{1}{6}\pi t) + 4\sin^2(\frac{1}{6}\pi t) + 9\cos^2(2\pi t) - 12\cos(2\pi t)\cos(\frac{1}{6}\pi t) + 4\cos^2(\frac{1}{6}\pi t)}$

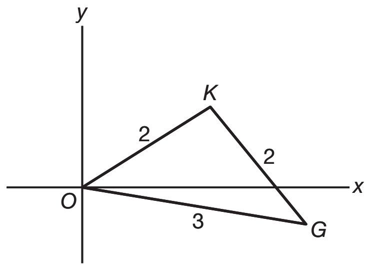
$= \sqrt{9(\sin^2(2\pi t) + \cos^2(2\pi t)) + 4(\sin^2(\frac{1}{6}\pi t) + \cos^2(\frac{1}{6}\pi t)) - 12\sin(2\pi t)\sin(\frac{1}{6}\pi t) - 12\cos(2\pi t)\cos(\frac{1}{6}\pi t)}$

$= \sqrt{9 + 4 - 12(\cos(2\pi t)\cos(\frac{1}{6}\pi t) + \sin(2\pi t)\sin(\frac{1}{6}\pi t))}$

$= \sqrt{13 - 12\cos(2\pi t - \frac{1}{6}\pi t)} = \sqrt{13 - 12\cos(\frac{11}{6}\pi t)}$

d





Een gelijkbenige driehoek als

afstand = 3 $\vee$ afstand = 2

$\sqrt{13 - 12\cos(\frac{11}{6}\pi t)} = 3 \quad \vee \quad \sqrt{13 - 12\cos(\frac{11}{16}\pi t)} = 2$

$13 - 12\cos(\frac{11}{6}\pi t) = 9 \quad \vee \quad 13 - 12\cos(\frac{11}{6}\pi t) = 4$

$\cos(\frac{11}{6}\pi t) = \frac{1}{3} \quad \vee \quad \cos(\frac{11}{6}\pi t) = \frac{3}{4}$

Het eerste moment na $t = 0$ volgt uit $\cos(\frac{11}{6}\pi t) = \frac{3}{4}$

$\frac{11}{6}\pi t \approx 0{,}723$

$t \approx 0{,}125$

Dus op $t \approx 0{,}125$.

bladzijde 172

38 **a** $f(x) = \sin(x)$, dus $T(\frac{1}{2}\pi, 1)$ en $A(\pi, 0)$.

$g(x) = -\dfrac{4}{\pi^2}x(x - \pi)$

$g(0) = 0$, dus de grafiek van g gaat door O.

$g(\frac{1}{2}\pi) = -\dfrac{4}{\pi^2} \cdot \frac{1}{2}\pi \cdot \frac{1}{2}\pi = 1$, dus de grafiek van g gaat door T.

$g(\pi) = 0$, dus de grafiek van g gaat door A.

b $f(x) = \sin(x)$ geeft $f'(x) = \cos(x)$

$g(x) = -\dfrac{4}{\pi^2}x(x - \pi) = -\dfrac{4}{\pi^2}x^2 + \dfrac{4}{\pi}x$ geeft $g'(x) = -\dfrac{8}{\pi^2}x + \dfrac{4}{\pi}$

$$\left.\begin{array}{l} g'(0) = \dfrac{4}{\pi} \\ f'(0) = \cos(0) = 1 \end{array}\right\} \; g'(0) > f'(0)$$

c $\int_0^{\pi}(g(x) - f(x))\,dx = 0$ geeft $\int_0^{\pi}(ax(x - \pi) - \sin(x))\,dx = 0$

$\int_0^{\pi}(ax^2 - a\pi x - \sin(x))\,dx = 0$

$[\frac{1}{3}ax^3 - \frac{1}{2}a\pi x^2 + \cos(x)]_0^{\pi} = 0$

$\frac{1}{3}a\pi^3 - \frac{1}{2}a\pi^3 + \cos(\pi) - (0 - 0 + \cos(0)) = 0$

$-\frac{1}{6}a\pi^3 - 1 - 1 = 0$

$-\frac{1}{6}a\pi^3 = 2$

$a = -\dfrac{12}{\pi^3}$