1.6 Diagnostische toets

bladzijde 42

1

a $3x^2 - x = 0$
$x(3x - 1) = 0$
$x = 0 \lor 3x = 1$
$x = 0 \lor x = \frac{1}{3}$

b $3x^2 - 9x = 12$
$3x^2 - 9x - 12 = 0$
$x^2 - 3x - 4 = 0$
$(x + 1)(x - 4) = 0$
$x = -1 \lor x = 4$

c $3x^2 - x = 2$
$3x^2 - x - 2 = 0$
$D = (-1)^2 - 4 \cdot 3 \cdot -2 = 25$,
dus $\sqrt{D} = \sqrt{25} = 5$
$x = \frac{1-5}{6} = -\frac{2}{3} \lor x = \frac{1+5}{6} = 1$

d $x^2 + 14 = 16$
$x^2 = 2$
$x = \sqrt{2} \lor x = -\sqrt{2}$

e $(2x - 3)^2 = 81$
Stel $2x - 3 = p$.
$p^2 = 81$
$p = 9 \lor p = -9$
$2x - 3 = 9 \lor 2x - 3 = -9$
$2x = 12 \lor 2x = -6$
$x = 6 \lor x = -3$

f $(3x + 2)(x - 1) = 0$
$3x = -2 \lor x = 1$
$x = -\frac{2}{3} \lor x = 1$

g $x^2 = 7x + 13$
$x^2 - 7x - 13 = 0$
$D = (-7)^2 - 4 \cdot 1 \cdot -13 = 101$
$x = \frac{7-\sqrt{101}}{2} \lor x = \frac{7+\sqrt{101}}{2}$

h $(3x + 2)(x - 1) = (x + 5) \cdot x$
$3x^2 - 3x + 2x - 2 = x^2 + 5x$
$2x^2 - 6x - 2 = 0$
$x^2 - 3x - 1 = 0$
$D = (-3)^2 - 4 \cdot 1 \cdot -1 = 13$
$x = \frac{3-\sqrt{13}}{2} \lor x = \frac{3+\sqrt{13}}{2}$

i $(x + 2)^2 = 3x + 7$
$x^2 + 4x + 4 = 3x + 7$
$x^2 + x - 3 = 0$
$D = 1^2 - 4 \cdot 1 \cdot -3 = 13$
$x = \frac{-1-\sqrt{13}}{2} \lor x = \frac{-1+\sqrt{13}}{2}$

j $(x - 3)^2 - (x + 1)^2 = (x - 4)^2$
$x^2 - 6x + 9 - (x^2 + 2x + 1) = x^2 - 8x + 16$
$x^2 - 6x + 9 - x^2 - 2x - 1 = x^2 - 8x + 16$
$-x^2 = 8$
$x^2 = -8$
geen oplossingen

2

a $D = 4^2 - 4 \cdot 2 \cdot p = 16 - 8p$
geen oplossingen als $D < 0$
$16 - 8p < 0$
$-8p < -16$
$p > 2$

b $D = p^2 - 4 \cdot 3 \cdot 27 = p^2 - 324$
twee oplossingen als $D > 0$
$p^2 - 324 > 0$
$p^2 > 324$
$p < -18 \lor p > 18$

c $p = 0$ geeft $-6x + 12 = 0$
$-6x = -12$
$x = 2$

Voor $p \neq 0$ is $D = (-6)^2 - 4 \cdot p \cdot 12 = 36 - 48p$.
één oplossing als $D = 0$
$36 - 48p = 0$
$-48p = -36$
$p = \frac{3}{4}$

$p = \frac{3}{4}$ geeft $\frac{3}{4}x^2 - 6x + 12 = 0$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

$x = 4$

Voor $p = 0$ is de oplossing $x = 2$ en voor $p = \frac{3}{4}$ is de oplossing $x = 4$.

3 **a** $x = 2$ invullen geeft $2^2 + 4 \cdot 2 + p = 0$

$4 + 8 + p = 0$

$p = -12$

De vergelijking is $x^2 + 4x - 12 = 0$

$(x - 2)(x + 6) = 0$

$x = 2 \vee x = -6$

Dus $p = -12$ en de andere oplossing is $x = -6$.

b $p = 0$ geeft $2x + 5 = 0$, dus één oplossing.

Voor $p \neq 0$ is $D = 2^2 - 4 \cdot p \cdot 5 = 4 - 20p$.
twee oplossingen als $D > 0$

$4 - 20p > 0$

$-20p > -4$

$p < \frac{1}{5}$

De vergelijking heeft twee oplossingen voor $p < 0 \vee 0 < p < \frac{1}{5}$.

4 **a** $3x^3 + 5 = 86$

$3x^3 = 81$

$x^3 = 27$

$x = 3$

b $5x^4 - 6 = 9$

$5x^4 = 15$

$x^4 = 3$

$x = \sqrt[4]{3} \vee x = -\sqrt[4]{3}$

c $2x^3 + 19 = 5$

$2x^3 = -14$

$x^3 = -7$

$x = \sqrt[3]{-7}$

d $\frac{1}{2}(x + 2)^4 = \frac{1}{32}$

$(x + 2)^4 = \frac{1}{16}$

$x + 2 = \frac{1}{2} \vee x + 2 = -\frac{1}{2}$

$x = -1\frac{1}{2} \vee x = -2\frac{1}{2}$

e $100 - (2x + 1)^5 = 68$

$-(2x + 1)^5 = -32$

$(2x + 1)^5 = 32$

$2x + 1 = 2$

$2x = 1$

$x = \frac{1}{2}$

f $(2x + 4)^3 = 10$

$2x + 4 = \sqrt[3]{10}$

$2x = -4 + \sqrt[3]{10}$

$x = -2 + \frac{1}{2} \cdot \sqrt[3]{10}$

5 **a** $x^4 - 6x^2 + 5 = 0$

Stel $x^2 = p$.

$p^2 - 6p + 5 = 0$

$(p - 1)(p - 5) = 0$

$p = 1 \vee p = 5$

$x^2 = 1 \vee x^2 = 5$

$x = 1 \vee x = -1 \vee x = \sqrt{5} \vee x = -\sqrt{5}$

b $5x^4 - 6x^2 + 1 = 0$

Stel $x^2 = p$.

$5p^2 - 6p + 1 = 0$

$D = (-6)^2 - 4 \cdot 5 \cdot 1 = 16$,

dus $\sqrt{D} = \sqrt{16} = 4$

$p = \dfrac{6 - 4}{10} = \frac{1}{5} \vee p = \dfrac{6 + 4}{10} = 1$

$p = \frac{1}{5} \vee p = 1$

$x^2 = \frac{1}{5} \vee x^2 = 1$

$x = \sqrt{\frac{1}{5}} \vee x = -\sqrt{\frac{1}{5}} \vee x = 1 \vee x = -1$

c $x^4 - 6x^3 + 5x^2 = 0$

$x^2(x^2 - 6x + 5) = 0$

$x^2(x - 1)(x - 5) = 0$

$x = 0 \vee x = 1 \vee x = 5$

d $x^3 + 6x^2 + 2x = 0$

$x(x^2 + 6x + 2) = 0$

$x = 0 \vee x^2 + 6x + 2 = 0$

$D = 6^2 - 4 \cdot 1 \cdot 2 = 28$

$x = 0 \vee x = \dfrac{-6 - \sqrt{28}}{2} \vee x = \dfrac{-6 + \sqrt{28}}{2}$

e $3x^6 + 3 = 10x^3$

$3x^6 - 10x^3 + 3 = 0$

Stel $x^3 = p$.

$3p^2 - 10p + 3 = 0$

$D = (-10)^2 - 4 \cdot 3 \cdot 3 = 64$, dus $\sqrt{D} = \sqrt{64} = 8$

$p = \dfrac{10 - 8}{6} = \frac{1}{3} \vee p = \dfrac{10 + 8}{6} = 3$

$x^3 = \frac{1}{3} \vee x^3 = 3$

$x = \sqrt[3]{\frac{1}{3}} \vee x = \sqrt[3]{3}$

f $x^8 + x^4 = 42$

$x^8 + x^4 - 42 = 0$

Stel $x^4 = p$.

$p^2 + p - 42 = 0$

$(p + 7)(p - 6) = 0$

$p = -7 \vee p = 6$

$x^4 = -7 \vee x^4 = 6$

$x = \sqrt[4]{6} \vee x = -\sqrt[4]{6}$

6 **a** $|x^2-4|=21$
$x^2-4=21 \vee x^2-4=-21$
$x^2=25 \vee x^2=-17$
$x=5 \vee x=-5$

b $|4x^3-5|=17$
$4x^3-5=17 \vee 4x^3-5=-17$
$4x^3=22 \vee 4x^3=-12$
$x^3=5\frac{1}{2} \vee x^3=-3$
$x=\sqrt[3]{5\frac{1}{2}} \vee x=\sqrt[3]{-3}$

7 **a** $\sqrt{3x+5}+1=5$
$\sqrt{3x+5}=4$
kwadrateren geeft
$3x+5=16$
$3x=11$
$x=3\frac{2}{3}$
$x=3\frac{2}{3}$ geeft $\sqrt{11+5}+1=5$ voldoet

b $3x=5\sqrt{x+4}$
kwadrateren geeft
$9x^2=25(x+4)$
$9x^2=25x+100$
$9x^2-25x-100=0$
$D=(-25)^2-4\cdot 9\cdot -100=4225$,
dus $\sqrt{D}=\sqrt{4225}=65$
$x=\frac{25-65}{18}=-2\frac{2}{9} \vee x=\frac{25+65}{18}=5$
$x=-2\frac{2}{9}$ geeft $-6\frac{2}{3}=5\sqrt{-2\frac{2}{9}+4}$
voldoet niet
$x=5$ geeft $15=5\cdot\sqrt{9}$ voldoet

c $x=\sqrt{x}+6$
$x-6=\sqrt{x}$
kwadrateren geeft
$(x-6)^2=x$
$x^2-12x+36=x$
$x^2-13x+36=0$
$(x-4)(x-9)=0$
$x=4 \vee x=9$
$x=4$ geeft $4=\sqrt{4}+6$ voldoet niet
$x=9$ geeft $9=\sqrt{9}+6$ voldoet

d $2x+3\sqrt{x}=2$
$3\sqrt{x}=2-2x$
kwadrateren geeft
$9x=(2-2x)^2$
$9x=4-8x+4x^2$
$-4x^2+17x-4=0$
$D=17^2-4\cdot -4\cdot -4=225$,
dus $\sqrt{D}=\sqrt{225}=15$
$x=\frac{-17-15}{-8}=4 \vee x=\frac{-17+15}{-8}=\frac{1}{4}$
$x=4$ geeft $8+3\sqrt{4}=2$ voldoet niet
$x=\frac{1}{4}$ geeft $\frac{1}{2}+3\sqrt{\frac{1}{4}}=2$ voldoet

8 **a** $x^3-189=20x\sqrt{x}$
$x^3-20x\sqrt{x}-189=0$
Stel $x\sqrt{x}=p$.
$p^2-20p-189=0$
$(p+7)(p-27)=0$
$p=-7 \vee p=27$
$x\sqrt{x}=-7 \vee x\sqrt{x}=27$
$x\sqrt{x}=-7$ heeft geen oplossing
$x\sqrt{x}=27$ kwadrateren geeft $x^3=729$
$x=9$
$x=9$ geeft $9\sqrt{9}=27$ voldoet

b $x^5+12=8x^2\cdot\sqrt{x}$
$x^5-8x^2\cdot\sqrt{x}+12=0$
Stel $x^2\cdot\sqrt{x}=p$.
$p^2-8p+12=0$
$(p-2)(p-6)=0$
$p=2 \vee p=6$
$x^2\cdot\sqrt{x}=2 \vee x^2\cdot\sqrt{x}=6$
kwadrateren geeft
$x^5=4 \vee x^5=36$
$x=\sqrt[5]{4} \vee x=\sqrt[5]{36}$
$x=\sqrt[5]{4}$ geeft $\left(\sqrt[5]{4}\right)^2\cdot\sqrt{\sqrt[5]{4}}=2$ voldoet
$x=\sqrt[5]{36}$ geeft $\left(\sqrt{\sqrt[5]{36}}\right)^2\cdot\sqrt{\sqrt[5]{36}}=2$ voldoet

9 **a** $\frac{6x-18}{x+1}=0$
$6x-18=0$
$6x=18$
$x=3$
voldoet

b $\frac{x^2-5x+6}{2x+4}=0$
$x^2-5x+6=0$
$(x-2)(x-3)=0$
$x=2 \vee x=3$
vold. vold.

c $\frac{3x-5}{x+1}=\frac{x+2}{x+1}$
$3x-5=x+2$
$2x=7$
$x=3\frac{1}{2}$
voldoet

d $\frac{x^2-4}{2x+1}=\frac{x^2-4}{x-4}$
$x^2-4=0 \vee 2x+1=x-4$
$x^2=4 \vee x=-5$
$x=2 \vee x=-2 \vee x=-5$
vold. vold. vold.

e $\frac{2x-1}{x+1}=\frac{x+3}{x-4}$
kruiselings vermenigvuldigen geeft
$(2x-1)(x-4)=(x+1)(x+3)$
$2x^2-8x-x+4=x^2+3x+x+3$
$x^2-13x+1=0$
$D=(-13)^2-4\cdot 1\cdot 1=165$
$x=\frac{13-\sqrt{165}}{2} \vee x=\frac{13+\sqrt{165}}{2}$
voldoet voldoet

f $\frac{2x^2-4}{x+5}=1\frac{3}{4}$
$\frac{2x^2-4}{x+5}=\frac{7}{4}$
kruiselings vermenigvuldigen geeft
$(2x^2-4)\cdot 4=(x+5)\cdot 7$
$8x^2-16=7x+35$
$8x^2-7x-51=0$
$D=(-7)^2-4\cdot 8\cdot -51=1681$,
dus $\sqrt{D}=\sqrt{1681}=41$
$x=\frac{7-41}{16}=-2\frac{1}{8} \vee x=\frac{7+41}{16}=3$
voldoet voldoet

10 **a** $\begin{cases} 4x+5y=27 \\ -2x+3y=25 \end{cases}$ $\begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 4x+5y=27 \\ -4x+6y=50 \end{cases}$ +
$11y=77$
$\left.\begin{array}{r} y=7 \\ -2x+3y=25 \end{array}\right\}$ $-2x+3\cdot 7=25$
$-2x+21=25$
$-2x=4$
$x=-2$
Dus $x=-2 \wedge y=7$.

b $\begin{cases} 2x+3y=7 \\ 5x-2y=8 \end{cases}$ $\begin{vmatrix} 2 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 4x+6y=14 \\ 15x-6y=24 \end{cases}$ +
$19x=38$
$\left.\begin{array}{r} x=2 \\ 5x-2y=8 \end{array}\right\}$ $5\cdot 2-2y=8$
$10-2y=8$
$-2y=-2$
$y=1$
Dus $x=2 \wedge y=1$.

11 (2, 18) invullen geeft $18=a\cdot 2^2+b\cdot 2$ dus $4a+2b=18$
(−4, 0) invullen geeft $0=a\cdot(-4)^2+b\cdot -4$ dus $16a-4b=0$
$\begin{cases} 4a+2b=18 \\ 16a-4b=0 \end{cases}$ $\begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 8a+4b=36 \\ 16a-4b=0 \end{cases}$ +
$24a=36$
$\left.\begin{array}{r} a=1\frac{1}{2} \\ 4a+2b=18 \end{array}\right\}$ $4\cdot 1\frac{1}{2}+2b=18$
$6+2b=18$
$2b=12$
$b=6$
Dus $y=1\frac{1}{2}x^2+6x$.

12 **a** Substitutie van $y=\frac{2}{3}x-4$ in $5x-3y=3$ geeft $5x-3(\frac{2}{3}x-4)=3$
$5x-2x+12=3$
$3x=-9$
$x=-3$
$x=-3$ geeft $y=\frac{2}{3}\cdot -3-4=-6$
Dus $x=-3 \wedge y=-6$.

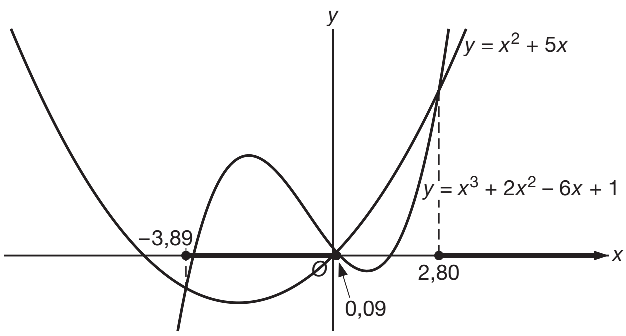
b Substitutie van $y = x^2 - 4x + 6$ in $2x + 3y = 10$ geeft

$$2x + 3(x^2 - 4x + 6) = 10$$
$$2x + 3x^2 - 12x + 18 = 10$$
$$3x^2 - 10x + 8 = 0$$
$$D = (-10)^2 - 4 \cdot 3 \cdot 8 = 4, \text{ dus } \sqrt{D} = \sqrt{4} = 2$$
$$x = \frac{10-2}{6} = 1\tfrac{1}{3} \vee x = \frac{10+2}{6} = 2$$

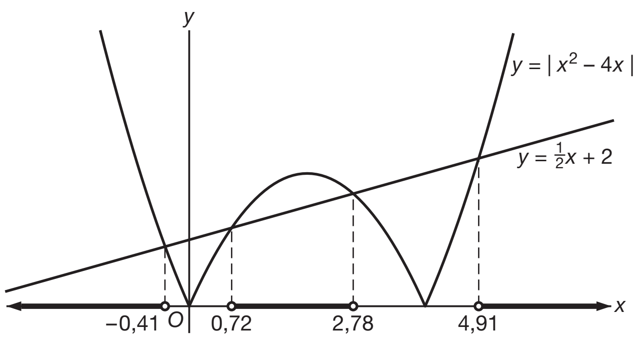
$x = 1\tfrac{1}{3}$ geeft $y = (1\tfrac{1}{3})^2 - 4 \cdot 1\tfrac{1}{3} + 6 = 2\tfrac{4}{9}$
$x = 2$ geeft $y = 2^2 - 4 \cdot 2 + 6 = 2$
Dus $(x = 1\tfrac{1}{3} \wedge y = 2\tfrac{4}{9}) \vee (x = 2 \wedge y = 2)$.

13 **a** Voer in $y_1 = x^4 - 4x^2$ en $y_2 = 0{,}5x - 2$.
De optie intersect geeft $x \approx -1{,}75 \vee x \approx -0{,}86 \vee x \approx 0{,}69 \vee x \approx 1{,}93$.
b Voer in $y_1 = \text{abs}(x^3 - 3x)$ en $y_2 = -\tfrac{1}{2}x + 2$.
De optie intersect geeft $x \approx -2{,}11 \vee x \approx 0{,}65 \vee x \approx 1{,}46 \vee x \approx 1{,}89$.

14 **a** Voer in $y_1 = x^2 + 5x$ en $y_2 = x^3 + 2x^2 - 6x + 1$.
De optie intersect geeft $x \approx -3{,}89$, $x \approx 0{,}09$ en $x \approx 2{,}80$.

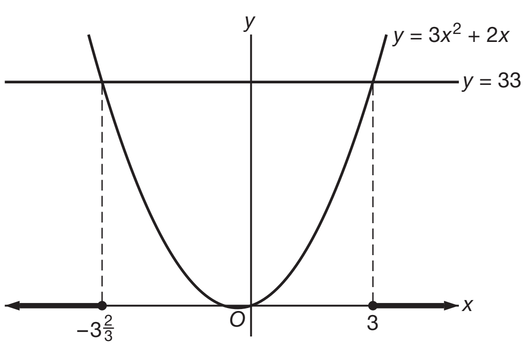


$x^2 + 5x \le x^3 + 2x^2 - 6x + 1$ geeft $-3{,}89 \le x \le 0{,}09 \vee x \ge 2{,}80$
b Voer in $y_1 = \text{abs}(x^2 - 4x)$ en $y_2 = \tfrac{1}{2}x + 2$.
De optie intersect geeft $x \approx -0{,}41$, $x \approx 0{,}72$, $x \approx 2{,}78$ en $x \approx 4{,}91$.



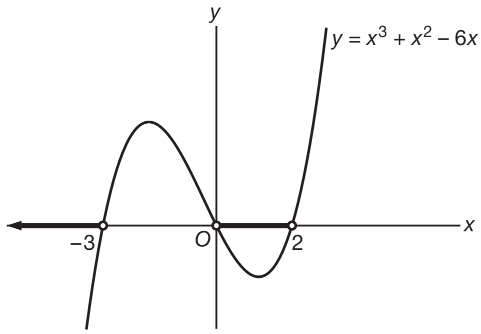
$|x^2 - 4x| > \tfrac{1}{2}x + 2$ geeft $x < -0{,}41 \vee 0{,}72 < x < 2{,}78 \vee x > 4{,}91$

15 **a** $3x^2 + 2x = 33$
$3x^2 + 2x - 33 = 0$
$D = 2^2 - 4 \cdot 3 \cdot -33 = 400$ dus $\sqrt{D} = \sqrt{400} = 20$
$$x = \frac{-2-20}{6} = -3\tfrac{2}{3} \vee x = \frac{-2+20}{6} = 3$$



$3x^2 + 2x \ge 33$ geeft $x \le -3\tfrac{2}{3} \vee x \ge 3$

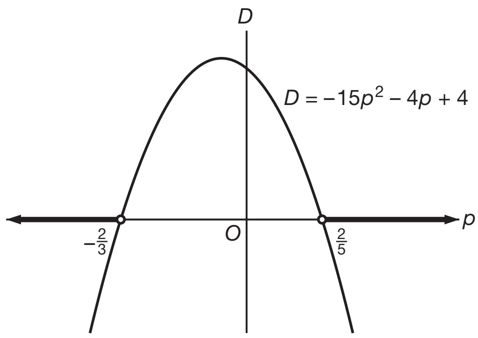
b $x^3 + x^2 - 6x = 0$
$x(x^2 + x - 6) = 0$
$x(x + 3)(x - 2) = 0$
$x = 0 \;\vee\; x = -3 \;\vee\; x = 2$



$x^3 + x^2 - 6x < 0$ geeft $x < -3 \;\vee\; 0 < x < 2$

16 **a** $p = 0$ geeft $-2x = 0$, dus één oplossing.
$px^2 + px - 2x + 4p = 0$
$px^2 + (p - 2)\,x + 4p = 0$
Voor $p \neq 0$ is $D = (p - 2)^2 - 4 \cdot p \cdot 4p = p^2 - 4p + 4 - 16p^2 = -15p^2 - 4p + 4.$
geen oplossingen als $D < 0$
} $-15p^2 - 4p + 4 < 0$

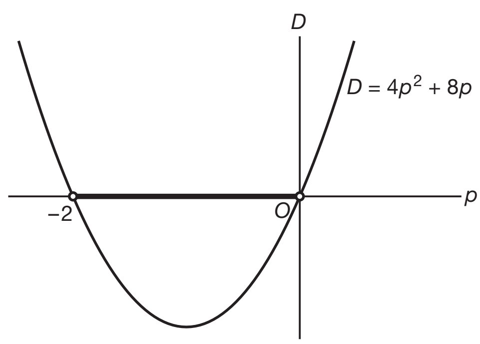
$-15p^2 - 4p + 4 = 0$
$D^* = (-4)^2 - 4 \cdot -15 \cdot 4 = 256$, dus $\sqrt{D^*} = \sqrt{256} = 16$
$p = \dfrac{4 - 16}{-30} = \frac{2}{5} \;\vee\; p = \dfrac{4 + 16}{-30} = -\frac{2}{3}$



De vergelijking heeft geen oplossingen voor $p < -\frac{2}{3} \;\vee\; p > \frac{2}{5}$.

b $p = 0$ geeft $-2x = 0$, dus één oplossing.
$px^3 + 2px^2 - 2x = 0$
$x\,(px^2 + 2px - 2) = 0$
$x = 0 \;\vee\; px^2 + 2px - 2 = 0$
Er is precies één oplossing als $px^2 + 2px - 2 = 0$ geen oplossingen heeft.
Voor $p \neq 0$ is $D = (2p)^2 - 4 \cdot p \cdot -2 = 4p^2 + 8p.$
geen oplossingen als $D < 0$
} $4p^2 + 8p < 0$

$4p^2 + 8p = 0$
$p(4p + 8) = 0$
$p = 0 \;\vee\; p = -2$



De vergelijking heeft precies één oplossing voor $-2 < p \leq 0$.